Oceanic Eddy-Induced Transport: Full-Tensor Approach

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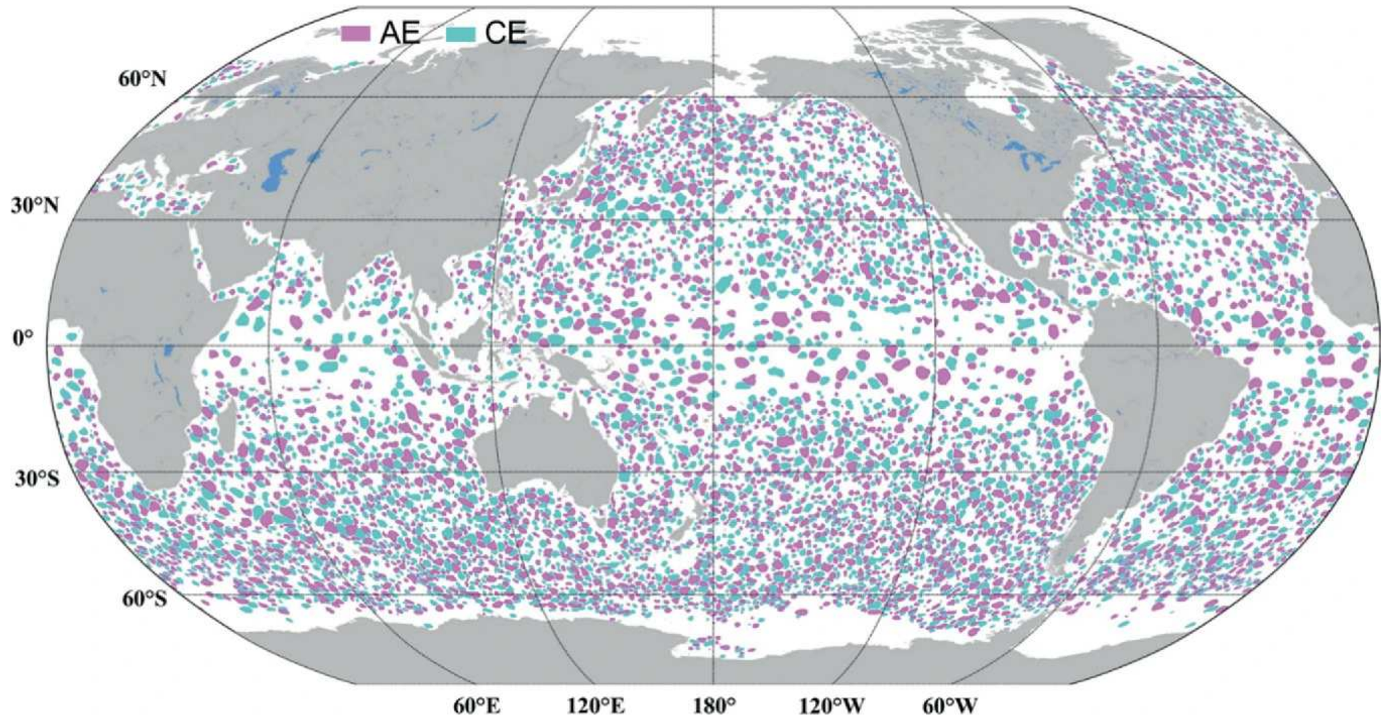
Results are published in a series of papers.

Mesoscale (Synoptic) Oceanic Eddies and their Diffusive Parameterization

Snapshot of the observed sea surface anomaly shows cyclonic and anticyclonic eddies all over the ocean



• Because eddies have important effects on the large-scale ocean circulation and climate dynamics, they have to be either *directly resolved*, which is computationally very expensive, or *parameterized*. Most parameterization approaches involve some *turbulent diffusion*.

• Turbulent diffusion is based on *flux-gradient relation* that replaces eddy flux (in large-scale dynamics for C) with large-scale gradient and involves *transport (tensor) coefficient* $\mathbf{K}$:

$$\overline{\mathbf{u}'C'} = -\mathbf{K}\cdot\nabla\overline{C} \qquad \Longrightarrow \qquad \frac{\partial\overline{C}}{\partial t} + \overline{\mathbf{u}}\cdot\nabla\overline{C} \;=\; \nabla\cdot\Big(\mathbf{K}\cdot\nabla\overline{C}\Big)\,,$$

where overbar and prime assume some scale decomposition: $C = \overline{C} + C'$, $\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$.

Statement of the Problem

• Our novel approach was to diagnose space-time dependent $\mathbf{K}(t, \mathbf{x})$ from an eddy-resolving model and **without imposing any constraints or assumptions**.

• We employed double-gyre quasigeostrophic (layered) ocean model that represents wind-driven midlatitude circulation with vigorous eddy activity.

• In each fluid layer we solve for the evolution of ensemble of mutually independent passive-tracer concentration fields. Both flow field and each tracer concentration solution were decomposed into the *large-scale* and *eddy* components by simple running-box filtering in space:

$$\mathbf{u}(t, \mathbf{x}) = \overline{\mathbf{u}}(t, \mathbf{x}) + \mathbf{u}'(t, \mathbf{x})\,, \qquad C(t, \mathbf{x}) = \overline{C}(t, \mathbf{x}) + C'(t, \mathbf{x})\,,$$

• Eddy effects were translated into 2×2 K-tensors that vary in space and time.

Double-Gyre Ocean Model

- Square basin of the North Atlantic size; flat bottom; β-plane; steady wind forcing; turbulent solutions; first baroclinic Rossby radius is 40 km; fine grid resolution (3.75 km).

- Governing equations for two-layer quasigeostrophic (QG) potential-vorticity (PV) model:

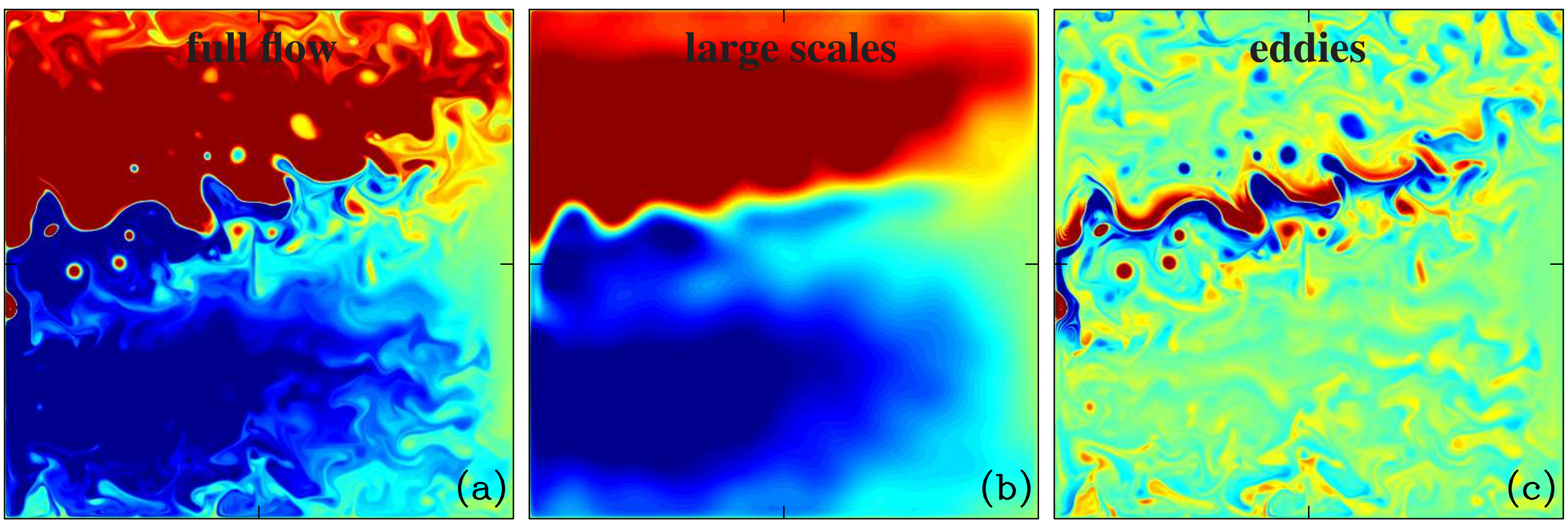
$$\frac{\partial q_1}{\partial t} + \mathbf{u}_1 \cdot \nabla q_1 + \beta v_1 = \nu \nabla^4 \psi_1 + W$$

$$\frac{\partial q_2}{\partial t} + \mathbf{u}_2 \cdot \nabla q_2 + \beta v_2 = \nu \nabla^4 \psi_2 - \gamma \nabla^2 \psi_2$$

$$u_i = -\frac{\partial \psi_i}{\partial y}, \qquad v_i = \frac{\partial \psi_i}{\partial x}$$

$$q_1 = \nabla^2 \psi_1 + S_1 (\psi_2 - \psi_1), \qquad q_2 = \nabla^2 \psi_2 + S_2 (\psi_1 - \psi_2)$$

Snapshot of the upper ocean PV anomaly and its large-scale and eddy components



Dynamics of Passive Tracers: Eddy Term

- Each tracer concentration $C(t, \mathbf{x})$ is governed by the corresponding conservation law:

$$\frac{\partial C}{\partial t} + \nabla \cdot (\mathbf{u} C) = \kappa \nabla^2 C + F \, .$$

- Decomposed fields are substituted back into each tracer equation:

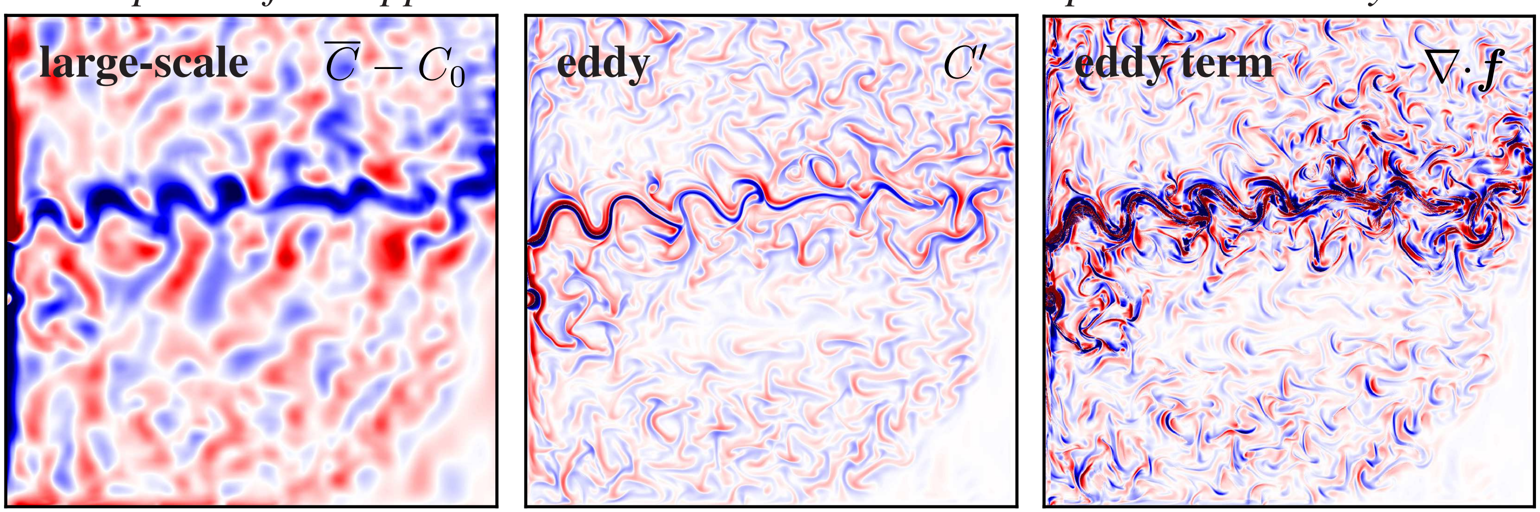
$$\frac{\partial \overline{C}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}\,\overline{C}) + \nabla \cdot (\overline{\mathbf{u}} C' + \mathbf{u}' \overline{C} + \mathbf{u}' C') = \kappa \nabla^2 \overline{C} + \kappa \nabla^2 C' - \frac{\partial C'}{\partial t} + F' + \overline{F} \, ,$$

where blue color indicates the *eddy term*.

- Non-advective rhs part of the eddy term can be represented as $-\nabla \cdot \mathbf{f}_n$ and absorbed into the tracer *eddy flux*:

$$\mathbf{f}(t, \mathbf{x}) = \overline{\mathbf{u}} C' + \mathbf{u}' \overline{C} + \mathbf{u}' C' + \mathbf{f}_n \qquad \Longrightarrow \qquad \boxed{\frac{\partial \overline{C}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}\,\overline{C}) + \nabla \cdot \mathbf{f} = \kappa \nabla^2 \overline{C} + \overline{F}}$$

Snapshot of the upper-ocean tracer concentration components and eddy term



Transport Tensor

• *Transport tensor* **K(t,x)** can be found from the assumed flux-gradient relation:

$$\boxed{\mathbf{f} = -\mathbf{K}\cdot\nabla\overline{C}}\,, \qquad \mathbf{K}(t,\mathbf{x}) = \begin{bmatrix} K_{11}(t,\mathbf{x}) & K_{12}(t,\mathbf{x}) \\ K_{21}(t,\mathbf{x}) & K_{22}(t,\mathbf{x}) \end{bmatrix}\,.$$

Since $\mathbf{K}$ has 4 unknowns, the relation is underdetermined. We resolved this problem by considering pairs of different tracers, e.g., C^p and C^q, and by solving the system of equations:

$$\begin{aligned} \mathbf{f}^p &= -\mathbf{K}\cdot\nabla\overline{C^p}\,, \\ \mathbf{f}^q &= -\mathbf{K}\cdot\nabla\overline{C^q}\,, \end{aligned}$$

under the assumption that $\mathbf{K}(t,\mathbf{x})$ is unique for both tracers (i.e., tracer-independent). Once $\mathbf{K}(t,\mathbf{x})$ is obtained, the eddy term becomes parameterized:

$$\frac{\partial\overline{C}}{\partial t} + \ldots = \nabla\cdot(\mathbf{K}\cdot\nabla\overline{C}) + \ldots\,.$$

• *Eddy flux reduction.* $\mathbf{K}(t,\mathbf{x})$ can be reduced by squeezing out (large and inert) rotational fluxes via the Helmholtz decomposition:

$$\mathbf{f} = \nabla\Phi + \nabla\times\Psi\,, \qquad \boxed{\nabla\cdot\mathbf{f} = \nabla^2\Phi}\,, \qquad \nabla\times\mathbf{f} = \nabla^2\Psi\,,$$

where $\nabla\Phi$ is the *divergent flux*, and $\nabla\times\Psi$ is the *rotational flux*.

• *Transport tensor decomposition.* **K** (i.e., its spatio-temporal maps) can be decomposed into its symmetric *diffusion S-tensor* and antisymmetric *advection A-tensor* components:

$$\mathbf{K} = \mathbf{S} + \mathbf{A}\,, \qquad \mathbf{S} = \begin{bmatrix} S_{11}(t,\mathbf{x}) & S_{12}(t,\mathbf{x}) \\ S_{12}(t,\mathbf{x}) & S_{22}(t,\mathbf{x}) \end{bmatrix}\,, \qquad \mathbf{A} = \begin{bmatrix} 0 & -A(t,\mathbf{x}) \\ A(t,\mathbf{x}) & 0 \end{bmatrix}\,.$$

• Diffusion tensor can be locally rotated by the *diffusion angle* $\alpha(t, \mathbf{x})$ until it is diagonalized with the *diffusion eigenvalues* λ_1 *and* λ_2 (these are 3 fundamental S-tensor properties):

$$\mathbf{S}_\alpha = \begin{bmatrix} \lambda_1(t, \mathbf{x}) & 0 \\ 0 & \lambda_2(t, \mathbf{x}) \end{bmatrix} .$$

• Advection tensor results in the flux divergence, which can be written as advection operator:

$$\nabla \cdot \mathbf{f}_{adv} = \frac{\partial A}{\partial x} \frac{\partial C}{\partial y} - \frac{\partial A}{\partial y} \frac{\partial C}{\partial x} = J(A, C) \,,$$

where A acts as the flux streamfunction. We introduce *eddy-induced velocity (EIV)*,

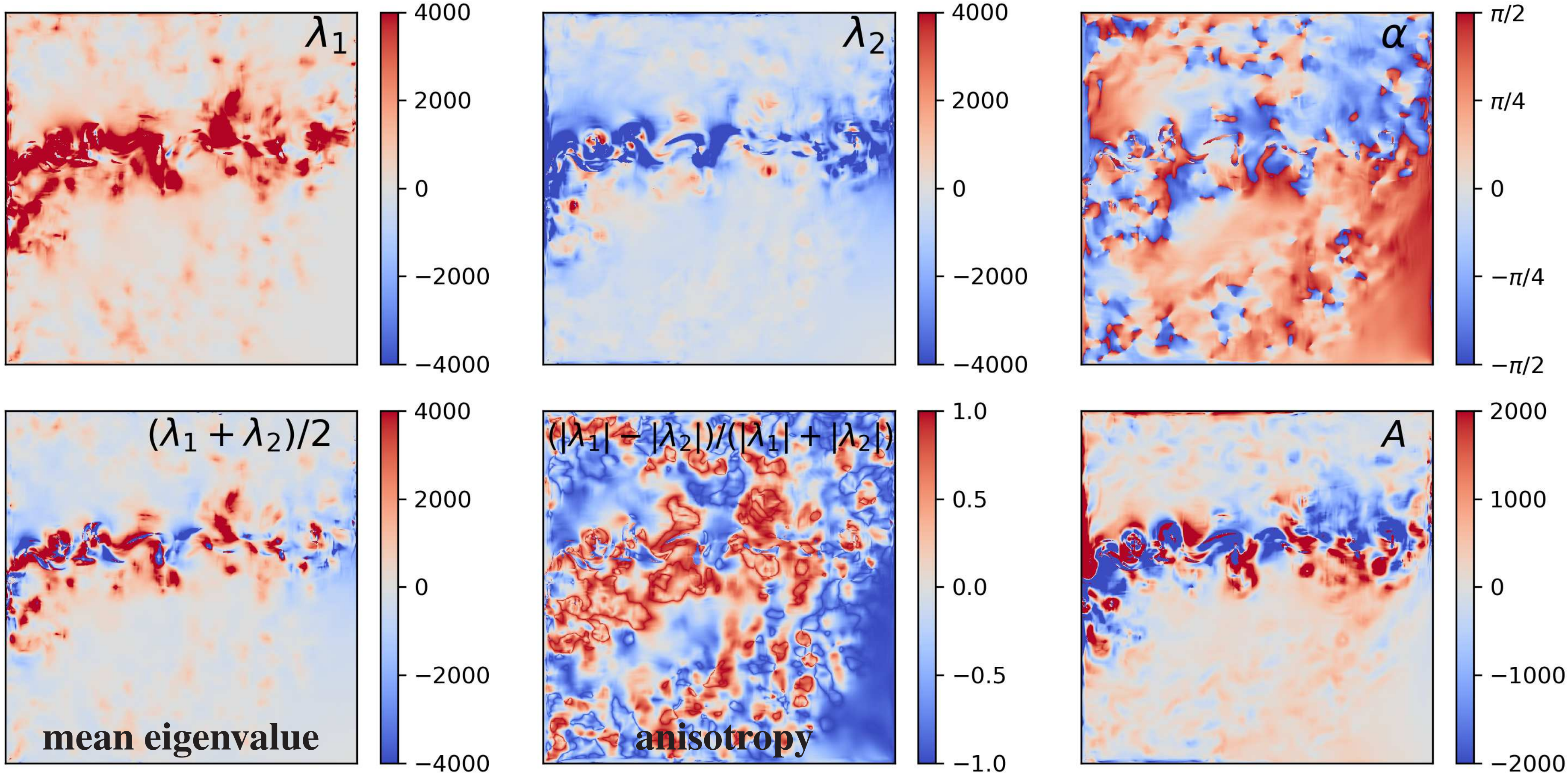
$$\mathbf{u}_*^c = \Big(- \frac{\partial A}{\partial y}, \frac{\partial A}{\partial x}\Big) \,,$$

which is fundamentally different from the bolus velocity used in Gent-McWilliams parameterization.

• *Fundamental elements* of $\mathbf{K}(t, \mathbf{x})$ are described by spatio-temporal maps of $\lambda_1,\ \lambda_2,\ \alpha,\ A$.

Transport Tensor: Illustration

Snapshot of the upper-ocean fundamental properties of K-tensor



• Note in the figure: (1) prevailing opposite polarity of eigenvalues, (2) large A-tensor, (3) large tensor rotations and anisotropy, (4) significant spatial inhomogeneity.

None of these feature are typically taken into account by eddy parameterizations!

Summary of Results

• The key aspect: *we imposed no constraints on* **K** *and explored it most completly*.

• We discovered robust *polar diffusion*, which is a tracer filamentation process characterized by co-existing diffusive and anti-diffusive eddy effects.

• We showed that diffusion is fundamentally insufficient for parameterization and there is *extra advection*.

• *Spatio-temporal variability* of K-tensor is significant, and this raises serious problem with its estimation from the available (mostly Lagrangian) ocean observations.

• *Question 1*: Which properties of transport K-tensor can be simplified for future parameterizations, and what are the consequences?

• *Question 2*: Should we keep going with flux-gradient relation or abandon/extend it?

• *Question 3*: Is transport K-tensor unique?

• *Many other related results are discussed in detail in the series of 7 publications: JFM, JFM, OM, OM, OM, GRL, JPO.*