Эмпирическая реконструкция режимов погоды средних широт как метастабильных состояний атмосферы

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Circulation of mid-latitude atmosphere

Time scales:

synoptic (weather), **intra- and inter-seasonal, interannual, decadal, etc.**

Low-frequency variability **(LFV)**

LFV modulates synoptic activity:

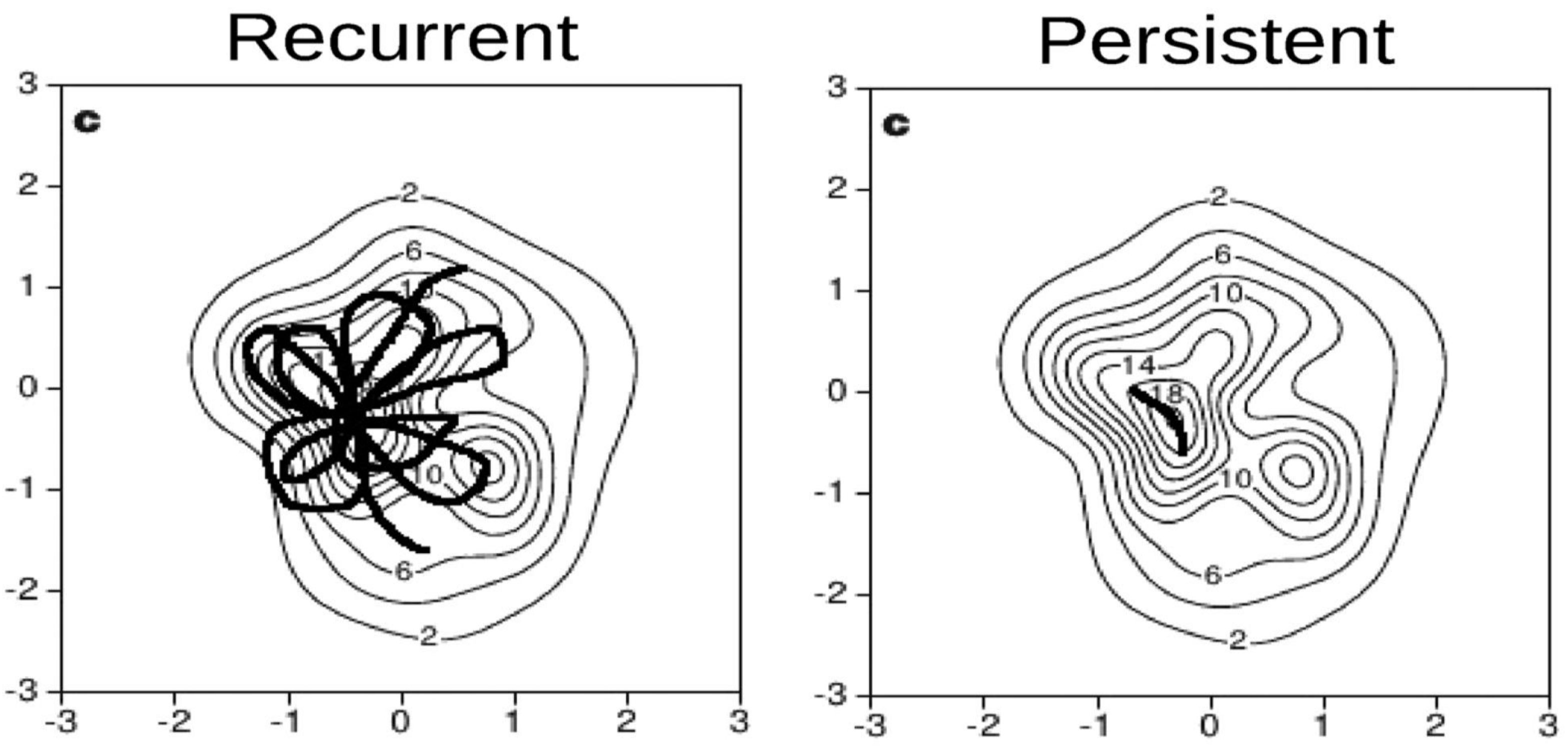
weather regimes – recurrent and persistence patterns of planetary scale

Metastable states in phase space → Recurrent persistent patterns (*weather regimes*)

Hannachi, A., et al. (2017). *Reviews of Geophysics*, *55*(1), 199–234. https://doi.org/10.1002/2015RG000509

Ghil, M., et al. (2019). *Sub-Seasonal to Seasonal Prediction*. Elsevier, 119–142. https://doi.org/10.1016/B978-0-12-811714-9.00006-1

Many works look for *recurrent* rather than *persistent* patterns



Hannachi, A., D. M. Straus, C. L. E. Franzke, S. Corti, and T. Woollings (2017), Low-frequency nonlinearity and regime behavior in the Northern Hemisphere extratropical atmosphere, *Rev. Geophys.*, *55*, 199–234, doi:10.1002/2015RG000509.

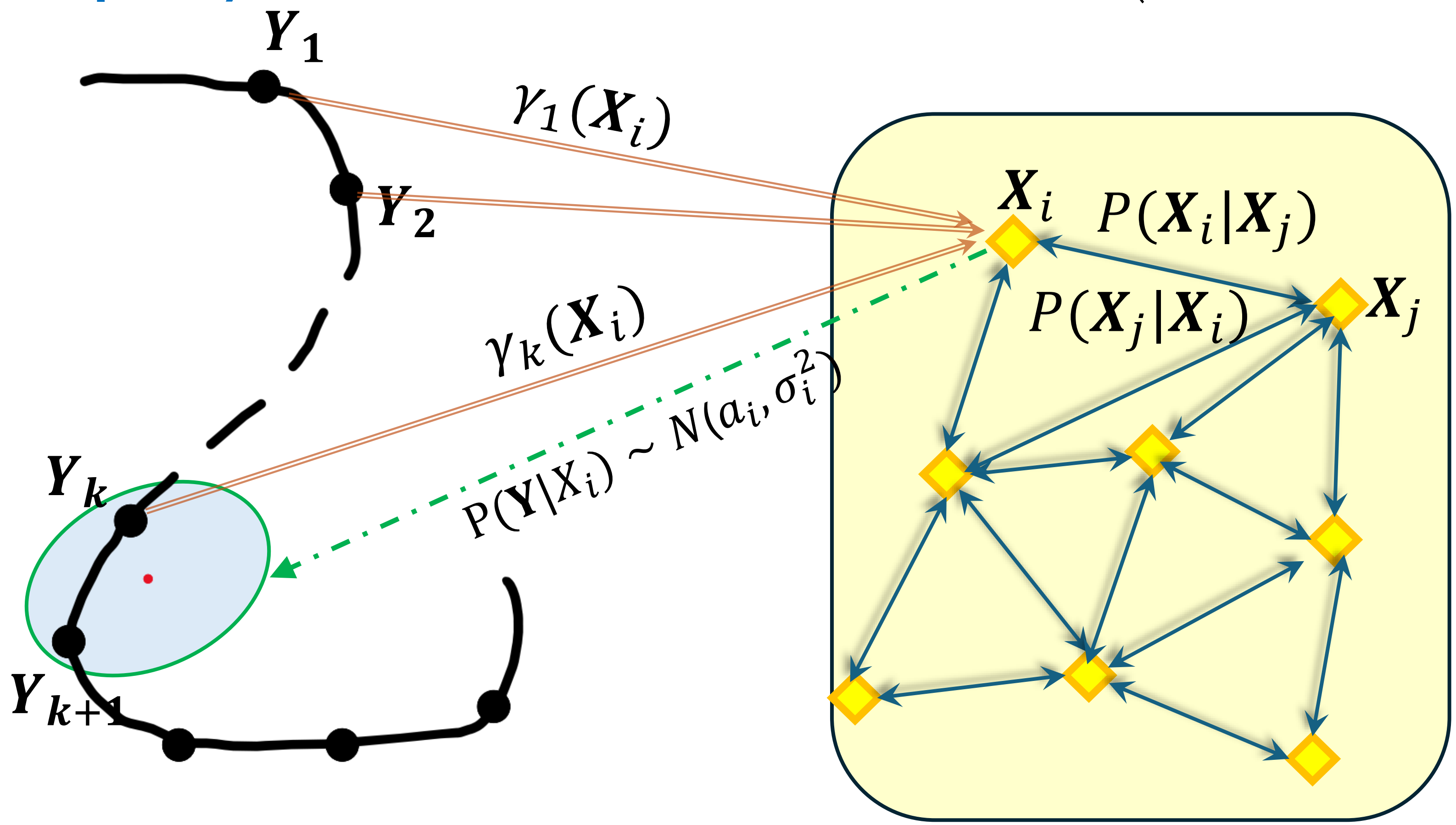
We need an evolution operator to find persistent (metastable) states

Hidden Markov model

Emission probabilities: $\mathrm{P}(\mathbf{Y}|X_i) \sim N(a_i, \sigma_i^2)$

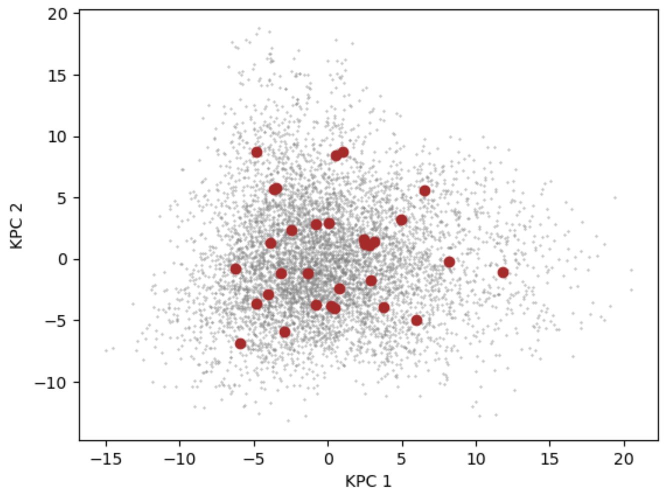
Dynamic variables
(state space)

Transition matrix $\mathbf{Q} = \begin{pmatrix} \mathrm{P}(X_1|X_1) & \cdots & \mathrm{P}(X_1|X_M) \\ \vdots & \ddots & \vdots \\ \mathrm{P}(X_M|X_1) & \cdots & \mathrm{P}(X_M|X_M) \end{pmatrix} \cdot \begin{pmatrix} \pi(X_1) \\ \vdots \\ \pi(X_M) \end{pmatrix} = \begin{pmatrix} \pi(X_1) \\ \vdots \\ \pi(X_M) \end{pmatrix}$



Hidden states
(discrete representation
of state space)

Hidden Markov model



Emission probabilities: $\mathrm{P}(\mathbf{Y}|X_i) \sim N(a_i, \sigma_i^2)$

Transition matrix **Q** = $\begin{pmatrix} \mathrm{P}(X_1|X_1) & \cdots & \mathrm{P}(X_1|X_M) \\ \vdots & \ddots & \vdots \\ \mathrm{P}(X_M|X_1) & \cdots & \mathrm{P}(X_M|X_M) \end{pmatrix} \cdot \begin{pmatrix} \pi(X_1) \\ \vdots \\ \pi(X_M) \end{pmatrix} = \begin{pmatrix} \pi(X_1) \\ \vdots \\ \pi(X_M) \end{pmatrix}$

Invariant distribution

$$\lim_{k \to \infty} \mathbf{Q}^k q = \pi$$

We use backward-forward algorithm to obtain Q,a, σ.
Also, it allows estimating occupation probabilities $\gamma_t(X_i)$ for each observation.

Hidden Markov model

Transition matrix **Q** = $\begin{pmatrix} P(X_1|X_1) & \cdots & P(X_1|X_M) \\ \vdots & \ddots & \vdots \\ P(X_M|X_1) & \cdots & P(X_M|X_M) \end{pmatrix} \cdot \begin{pmatrix} \pi(X_1) \\ \vdots \\ \pi(X_M) \end{pmatrix} = \begin{pmatrix} \pi(X_1) \\ \vdots \\ \pi(X_M) \end{pmatrix}$

"If the state space *S* of *X* can be decomposed in two or more sets with relatively infrequent transitions between those sets, the Markov chain is said to be metastable."

Franzke, C., D. Crommelin, A. Fischer, and A. J. Majda, 2008: A Hidden Markov Model Perspective on Regimes and Metastability in Atmospheric Flows. *J. Climate*, **21**, 1740–1757

Hidden Markov model

Metastable states

Idea: division of many states into communities, providing the greatest probability of remaining within the community

$$\begin{bmatrix} [A_1] & \cdot & \cdot & \cdot \\ \cdot & [A_2] & \cdot & \cdot \\ \cdot & \cdot & [\ddots] & \cdot \\ \cdot & \cdot & \cdot & [A_K] \end{bmatrix}$$

⇨ Reducing the transition matrix into a block matrix

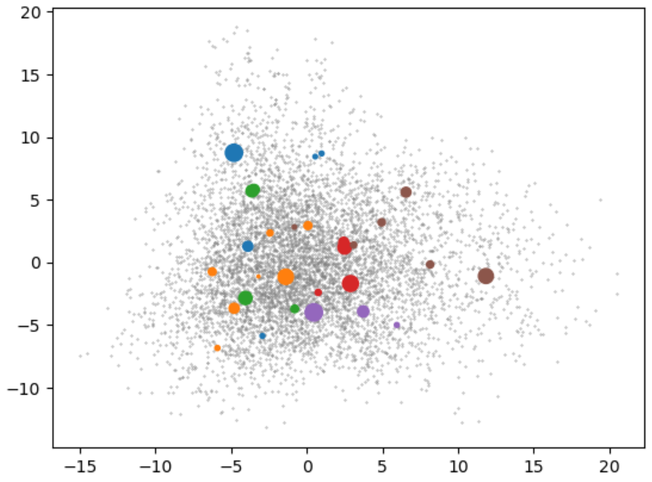
Finding the partition $\{A_k\}$ maximizing the quantity:

$$\langle P_Q(J \in A_k | J \in A_k) - P_{Q^\infty}(J \in A_k | J \in A_k) \rangle_{A_k}$$
$$= \sum_k \left[P_Q(J \in A_k | J \in A_k) - \pi(A_k) \right] \pi(A_k)$$

$$M = \sum_{A_k} \sum_{i,j \in A_k} [Q_{ij}\pi_j - \pi_i \pi_j] = \sum_{i,j} [Q_{ij}\pi_j - \pi_i \pi_j] g_{ij} = \sum_{i,j} B_{ij} g_{ij}$$

Hidden Markov model

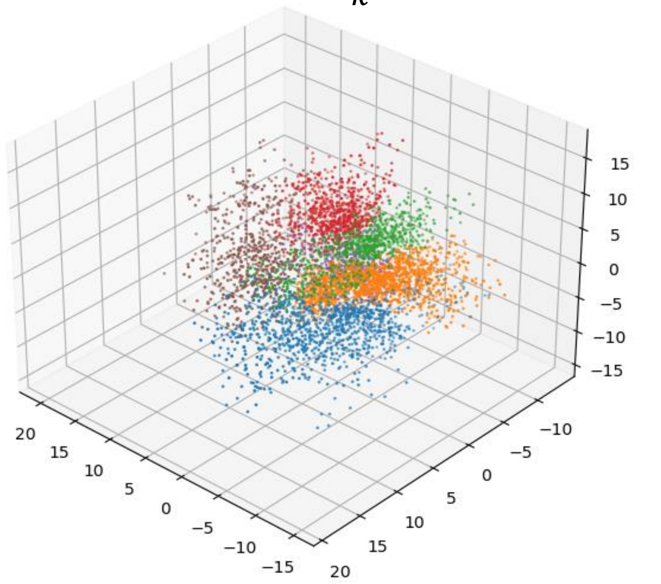
Finding metastable states



The matrix partition provides us with:

- a set of communities $\{A_k\}$ associated with regimes
- a matrix of transitions between the communities
- stability ranking of states inside a community
- probabilities of belonging of each observation to each community

$$\gamma_t(A_k) = \sum_{i \in A_k} \gamma_t(X_i)$$



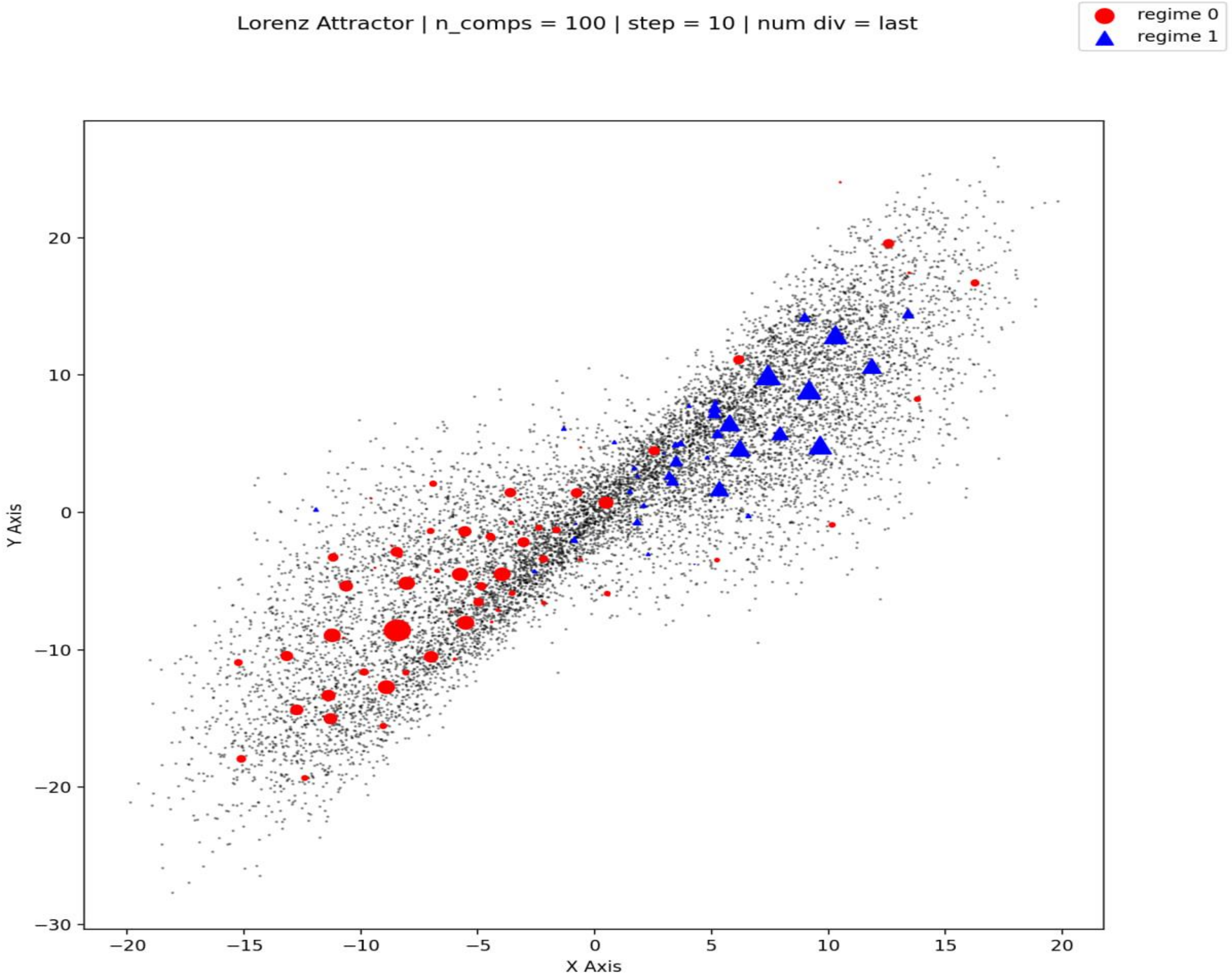
Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)

Hidden Markov model

Finding metastable states

Lorenz model

$$\dot{x} = 10(y - x),$$
$$\dot{y} = x(r - z) - y,$$
$$\dot{z} = xy - \frac{8}{3}z + s\xi.$$



Distance matrix $d_{ij} = d(\mathbf{x}_i, \mathbf{x}_j)$

Kernel PCA

$$\mathbf{K}_{ij} = \exp\left\{-\frac{d(\mathbf{x}_i,\ \mathbf{x}_j)^2}{2a^2}\right\}$$

Recurrence network

$$\mathbf{R}_{ij} = \begin{cases} 1, \mathbf{K}_{ij} > \varepsilon \\ 0, \text{otherwise} \end{cases}$$

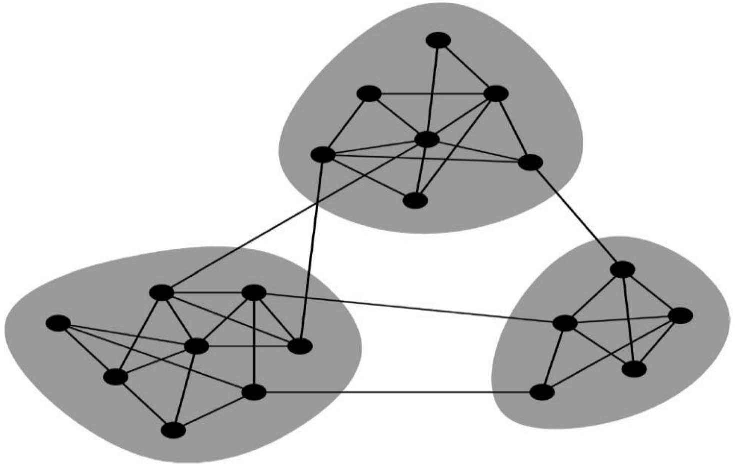
Dmitry Mukhin, Abdel Hannachi, Tobias Braun, and Norbert Marwan, **"Revealing recurrent regimes of mid-latitude atmospheric variability using novel machine learning method", Chaos 32, 113105 (2022)**

https://doi.org/10.1063/5.0109889

Mode detection:

recognizing communities of nodes, so that there are significantly more connections within communities than between them.

M. E. J. Newman. Modularity and community structure in networks. (2006)
PNAS, 103 (23) DOI: 10.1073/pnas.0601602103



Two basic steps:

1. Use KPCA to obtain a phase space projection that emphasizes the similarity of states.

2. Construct Markov chain on this subspace and determine its metastable states.

Hidden Markov model

Choosing time step

Transition matrix

$$\mathbf{Q}^k = \begin{pmatrix} \mathrm{P}(X_1|X_1) & \cdots & \mathrm{P}(X_1|X_M) \\ \vdots & \ddots & \vdots \\ \mathrm{P}(X_M|X_1) & \cdots & \mathrm{P}(X_M|X_M) \end{pmatrix}^k \cdot \begin{pmatrix} \pi(X_1) \\ \vdots \\ \pi(X_M) \end{pmatrix} = \begin{pmatrix} \pi(X_1) \\ \vdots \\ \pi(X_M) \end{pmatrix}$$

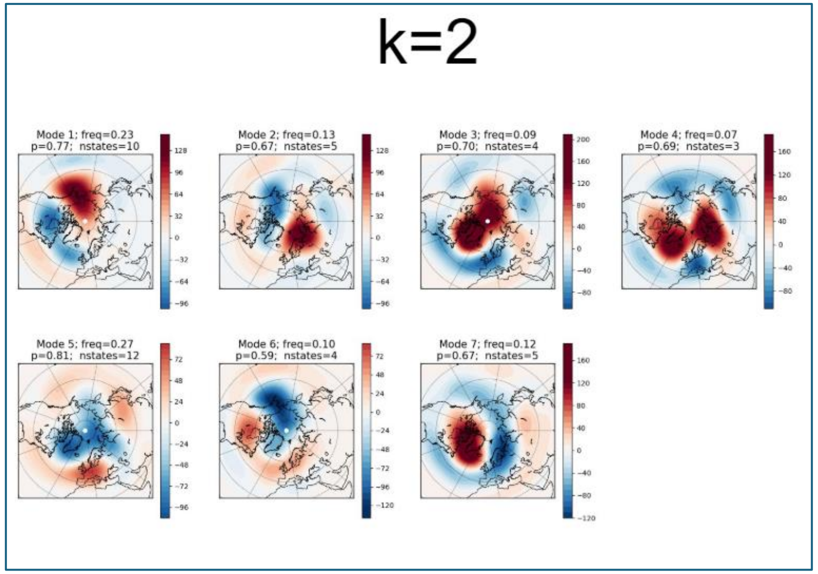
k is the time step

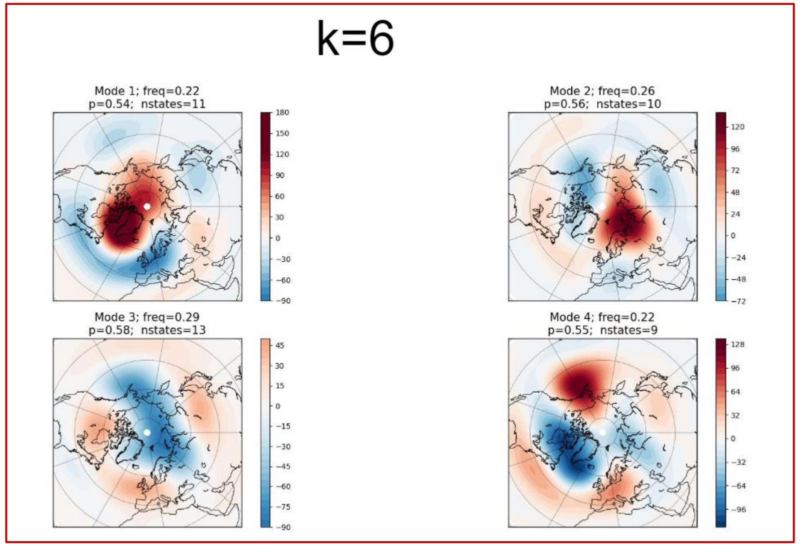
We use every data point for learning HMM, but look at the evolution across k steps

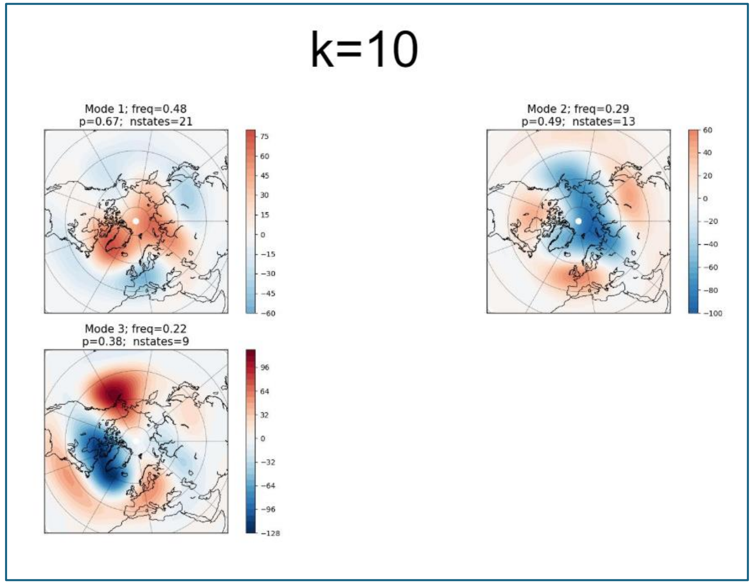
Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)

Hidden Markov model

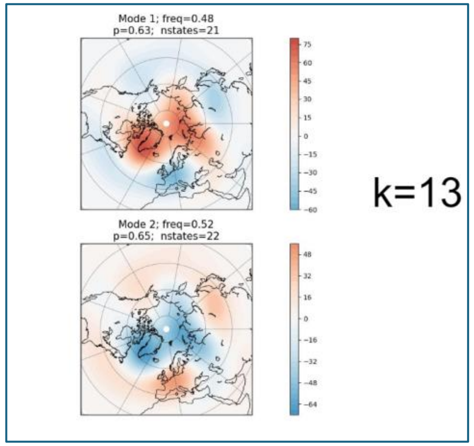
Choosing time step





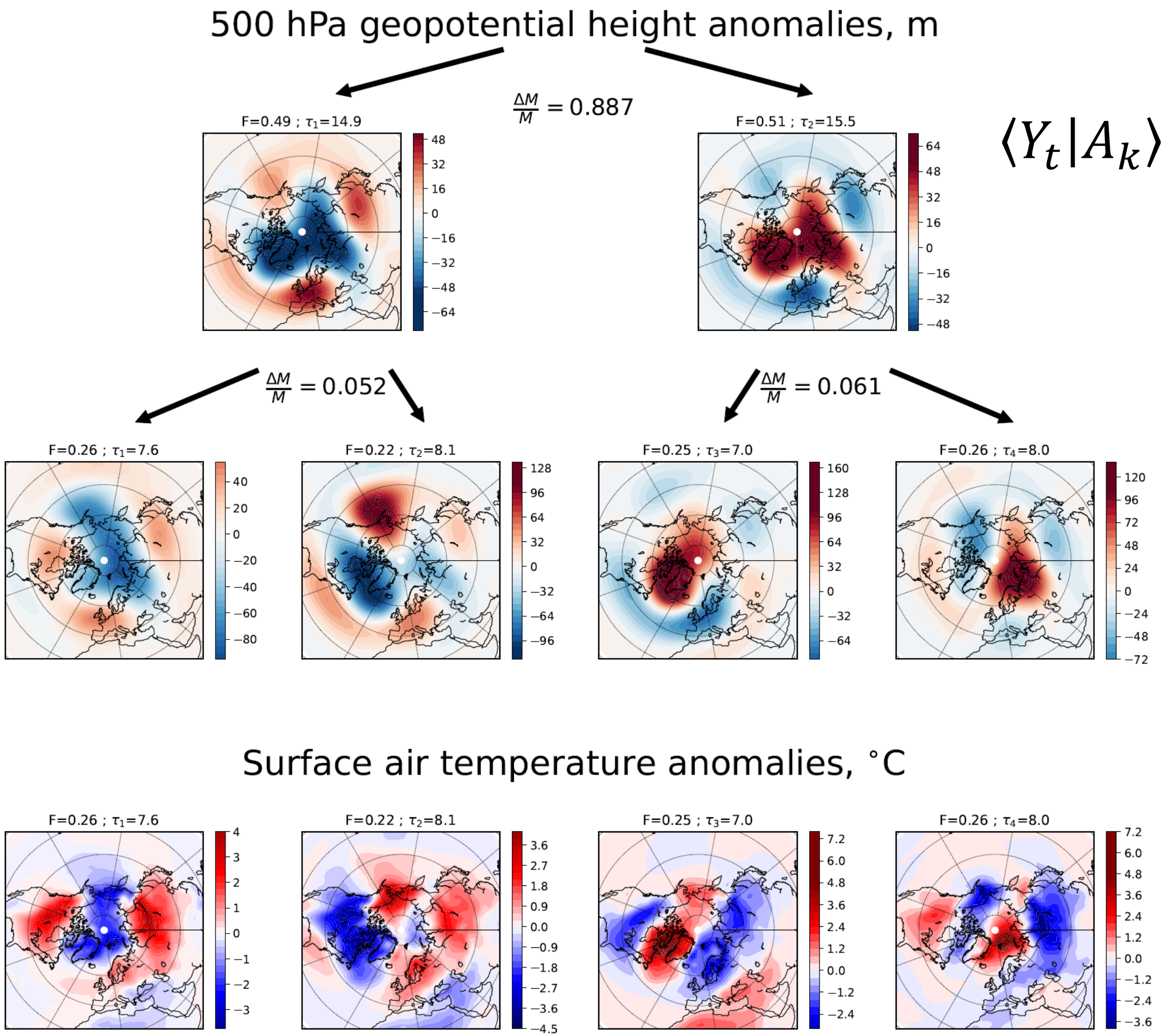


$$\langle Y_t | A_k \rangle = \frac{\sum_t Y_t \gamma_t(A_k)}{\sum_t \gamma_t(A_k)}$$



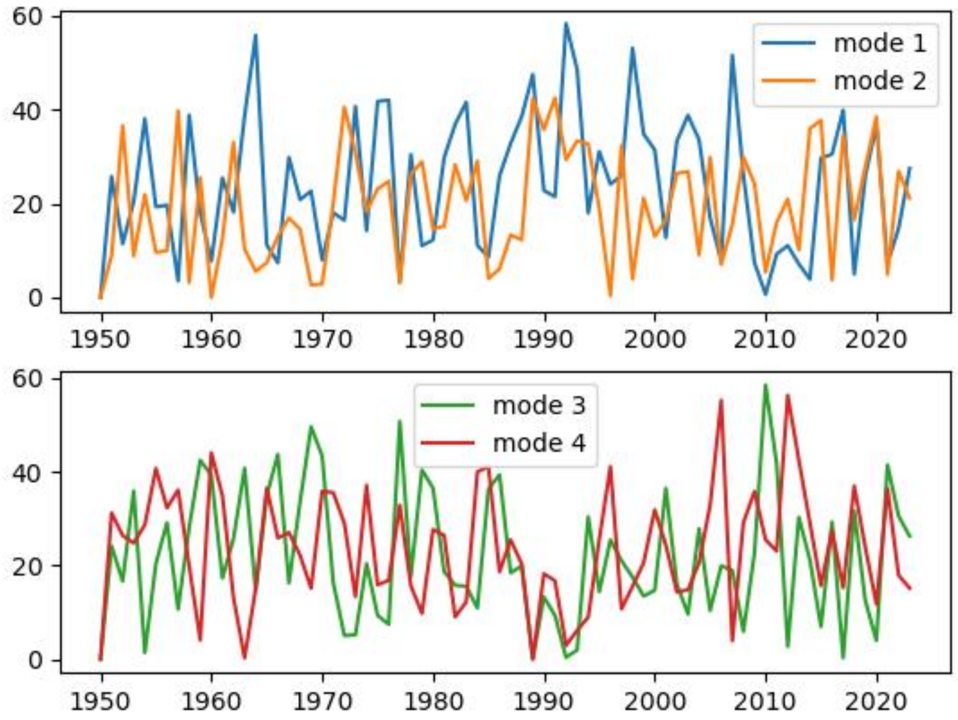
From $K = 6$ *the most stable results* are obtained

Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)

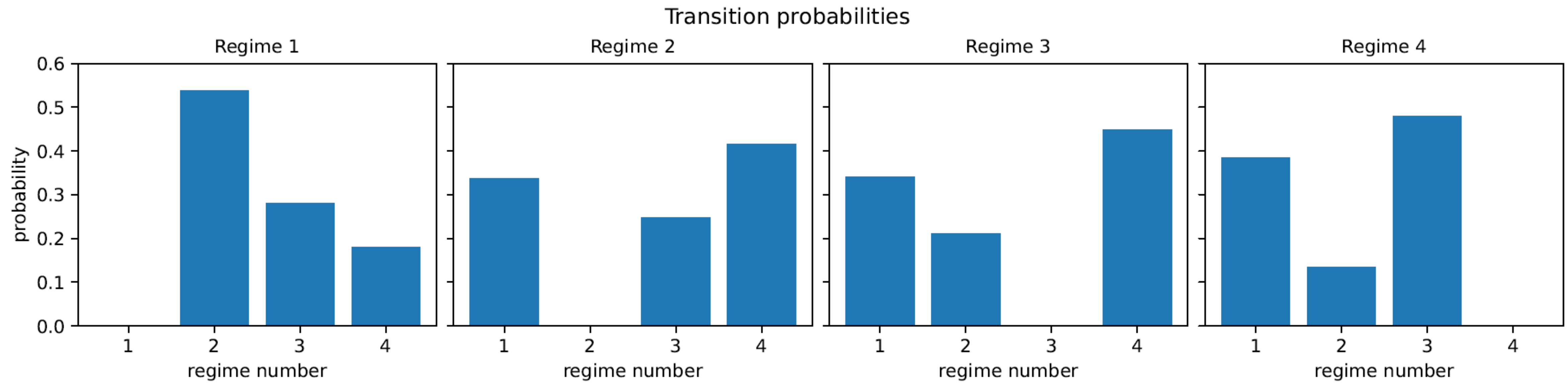


$$\langle Y_t | A_k \rangle = \frac{\sum_t Y_t \gamma_t(A_k)}{\sum_t \gamma_t(A_k)}$$

Regime frequencies (number of days per winter):

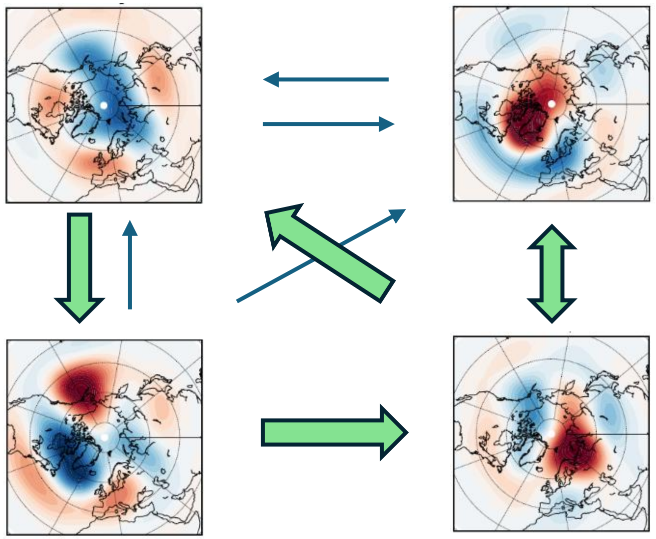


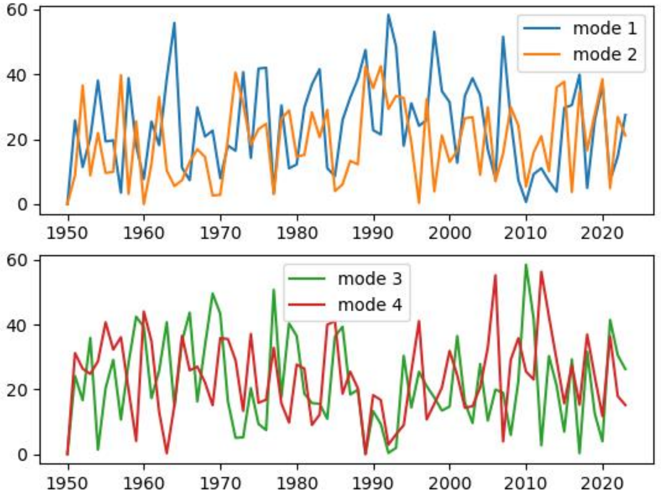
Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)



Reduced transition matrix (transitions between the regimes)

$$\overline{\mathbf{Q}^L} \approx \begin{pmatrix} 0.52 & 0.17 & 0.17 & 0.18 \\ 0.26 & 0.51 & 0.1 & 0.06 \\ 0.13 & 0.12 & 0.51 & 0.23 \\ 0.09 & 0.2 & 0.22 & 0.53 \end{pmatrix}$$





Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)

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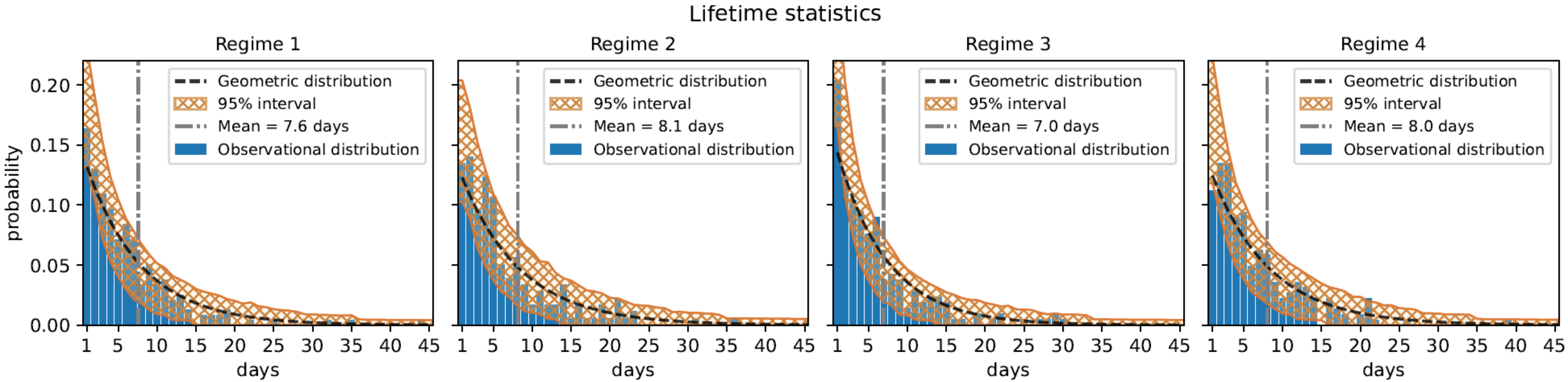


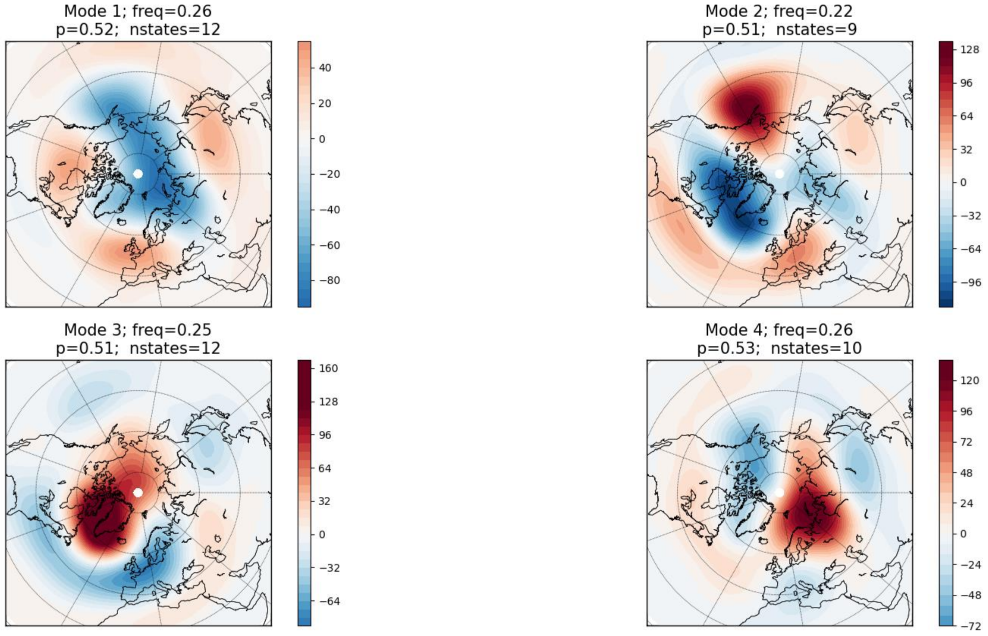
Geometric distribution

$$P(\tau) = q^{\tau}(1-q)^{\tau+1}$$

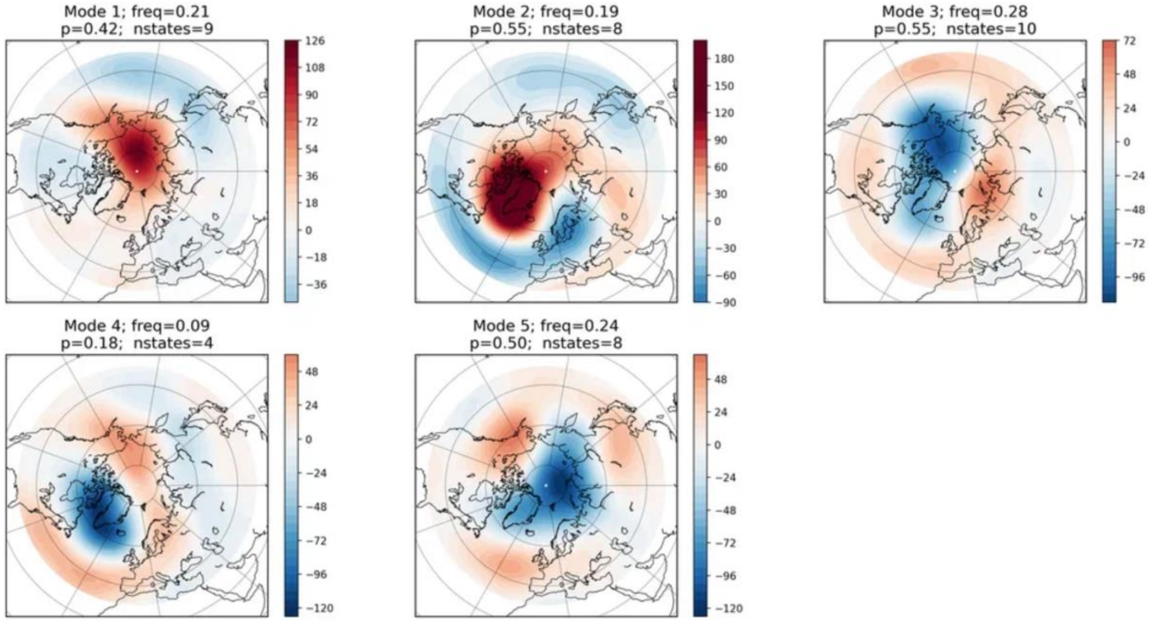
Open problems

- **Constructing full-year models. Interseasonal forecasting**
- **Forced and coupled Markov models. Studying interaction of regimes in different systems (e.g. stratosphere and troposphere, tropics and extratropics, etc.)**
- **Interection of regimes with Arctic sea ice dynamics**
- **Using non-Markovian data-driven models.**
  (Semyon Safonov's poster today about RNN–based model**)**
- **Regimes in future climate (based on CMIP projections)**
- … … …

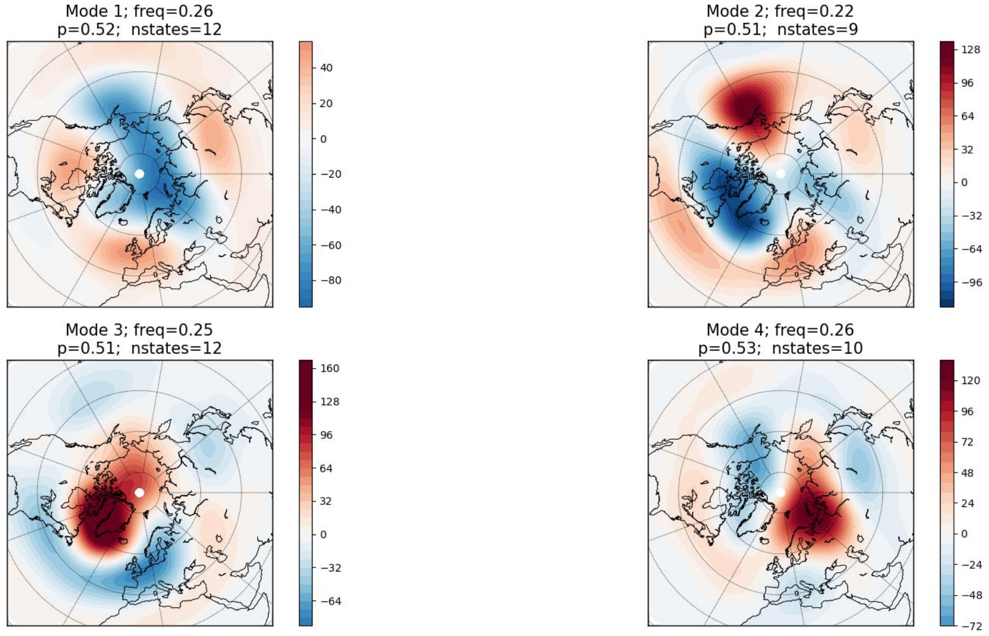
Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)



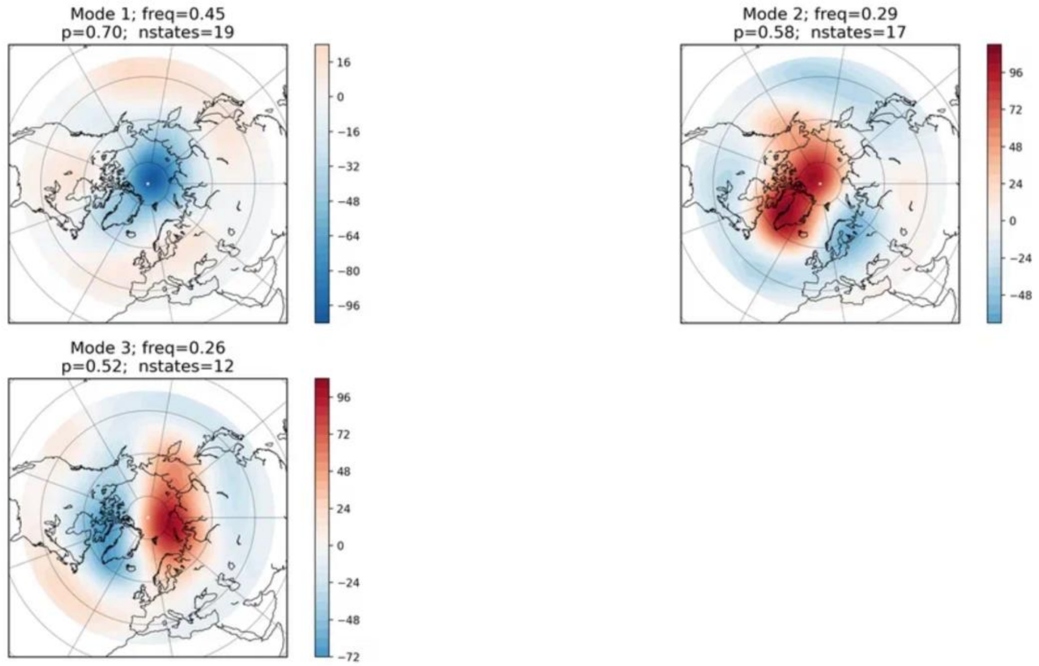
INMCM5 data: HGT-500, 1950-2014, winters (historical experiment)



Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)



INMCM5 data: HGT-500, 1950-2014, winters (preindustrial experiment)



Kernel PCA:

Constructing phase space projection based on similarity

Distance matrix $d_{ij} = d(\mathbf{x}_i, \mathbf{x}_j)$

↓

$$\mathbf{K}_{ij} = \exp\left\{-\frac{d(\mathbf{x}_i,\ \mathbf{x}_j)^2}{2a^2}\right\}$$

↓

Decomposing K into orthogonal basis:

$$\mathrm{K} = \sum_l \mathbf{P}_l \mathbf{P}_l^T + \bar{K}$$

$\bar{K} = K - K_C$ - *deviation from the centered K*

$\mathbf{P}_l$ is l*th* principal component

Clusters of similar states are well-separated in the space of a few leading component P

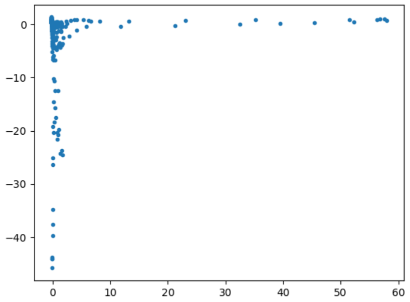
Gaussian kernels: $\mathbf{K}(\mathbf{x}_i, \mathbf{x}_j) = \exp\left\{-\frac{d(\mathbf{x}_i,\mathbf{x}_j)^2}{2a^2}\right\}$

Parameter a regulates nonlinearity of the transform. The larger a, the closer the transform to linear PCA:

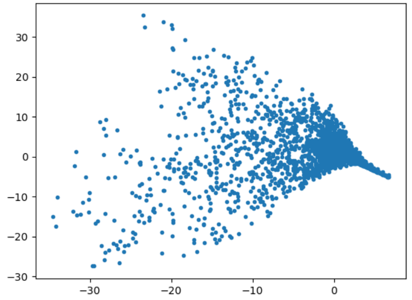
$$\mathbf{K}(\mathbf{x}_i, \mathbf{x}_j) = \exp\left\{-\frac{d(\mathbf{x}_i,\mathbf{x}_j)^2}{2a^2}\right\} \approx 1-\frac{d(\mathbf{x}_i,\mathbf{x}_j)^2}{2a^2} = 1-A(\mathbf{x}_i)-A(\mathbf{x}_j) + \frac{\mathbf{1}}{a^2}(\mathbf{x}_i, \mathbf{x}_j)$$

After centering: $\mathbf{K}(\mathbf{x}_i, \mathbf{x}_j) = \frac{\mathbf{1}}{a^2}(\mathbf{x}_i, \mathbf{x}_j)$

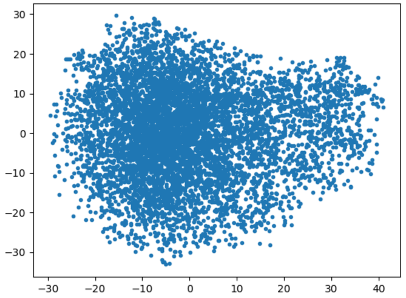
a =1



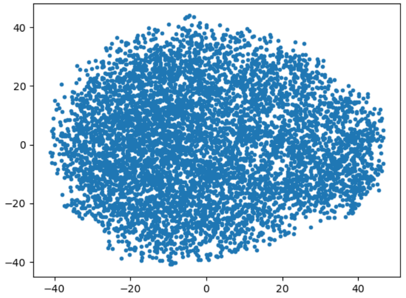
a =3



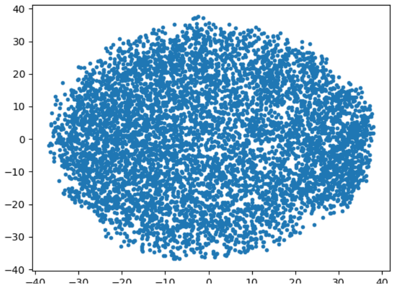
a =8



a =15

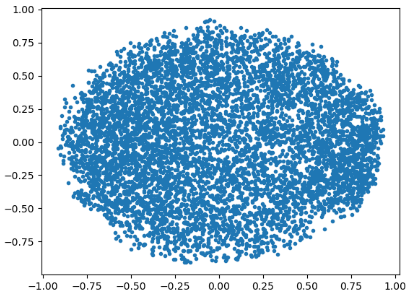


a =100



Reanalysis data: HGT-100, 1950-2023,
winters **(NCEP/NCAR reanalysis)**

EOF decomposition:



Nonlinear principal component analysis: Kernel PCA

Constructing a nonlinear embedding of the data space

$$\boldsymbol{\varphi}: \begin{pmatrix} x_1(t) \\ \vdots \\ x_D(t) \end{pmatrix} \rightarrow \begin{pmatrix} y_1(t) \\ \vdots \\ \vdots \\ y_M(t) \end{pmatrix}$$

⟹ Linear PCA in y-space

Temporal covariances (*dot products*) in a new M-dimensional space:

$$W_{ij} = \sum_{k=1}^{M} \varphi_k(\mathbf{x}(t_i))\varphi_k\left(\mathbf{x}(t_j)\right) = K(\mathbf{x}(t_i), \mathbf{x}(t_j)),\ \ i, j = 1, \cdots, N$$

Kernel function $\mathbf{K}(\mathbf{x_i}, \mathbf{x_j})$ – all we need to know for obtaining PCs (φ_k is unknown).

The kernels emphasizing similarity between two states (spatial patterns) $\mathbf{x_i} = \mathbf{x}(t_i), \mathbf{x_j} = \mathbf{x}(t_j)$:

Gaussian kernels: $\mathbf{K}(\mathbf{x_i}, \mathbf{x_j}) = \exp\left\{-\frac{d(\mathbf{x}_i,\mathbf{x}_j)^2}{2a^2}\right\}$

$d(\mathbf{x}_i, \mathbf{x}_j)$ – ***distance between the states***

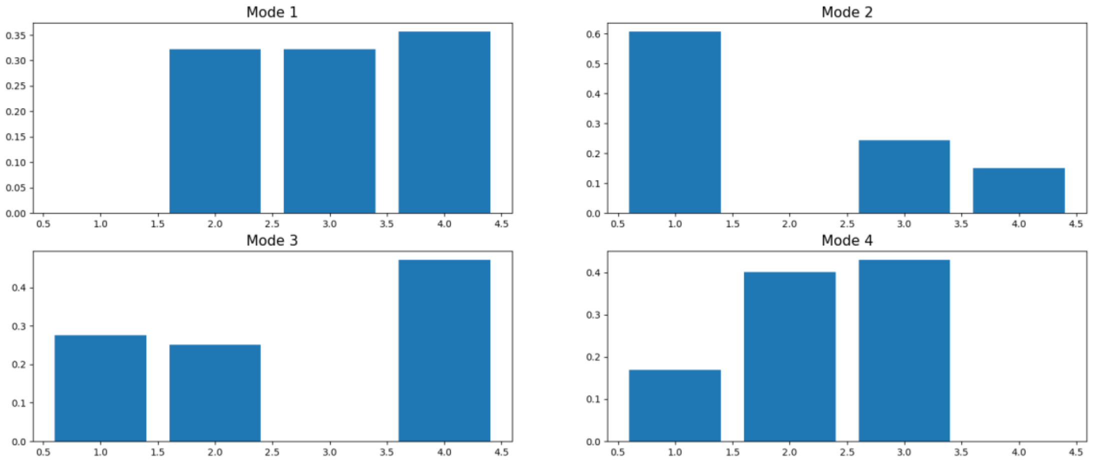
$$\mathbf{K}(\mathbf{x_i}, \mathbf{x_j}) = \sum_{k=0}^{\infty} \gamma_k \varphi_k(\mathbf{x_i})\varphi_k\left(\mathbf{x_j}\right)$$

Formally, we do the linear PCA in an infinite dimensional space

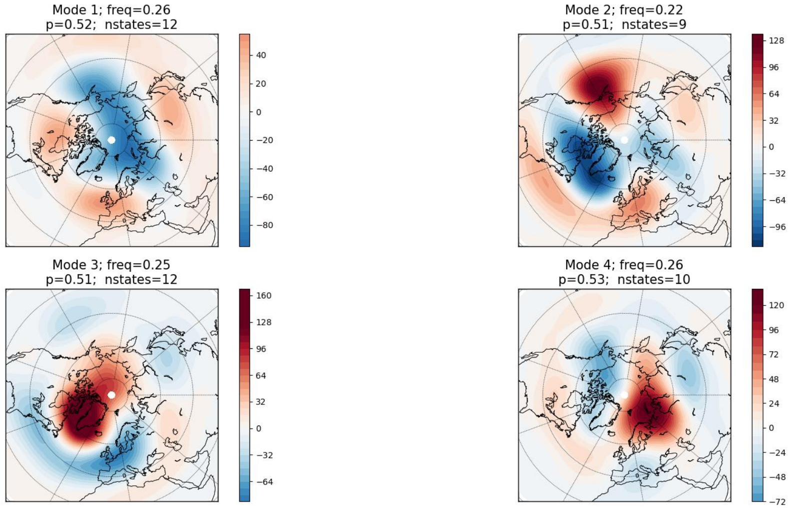
Hidden Markov model

Reanalysis data: HGT-500, 1950-2023,
winters (NCEP/NCAR reanalysis)

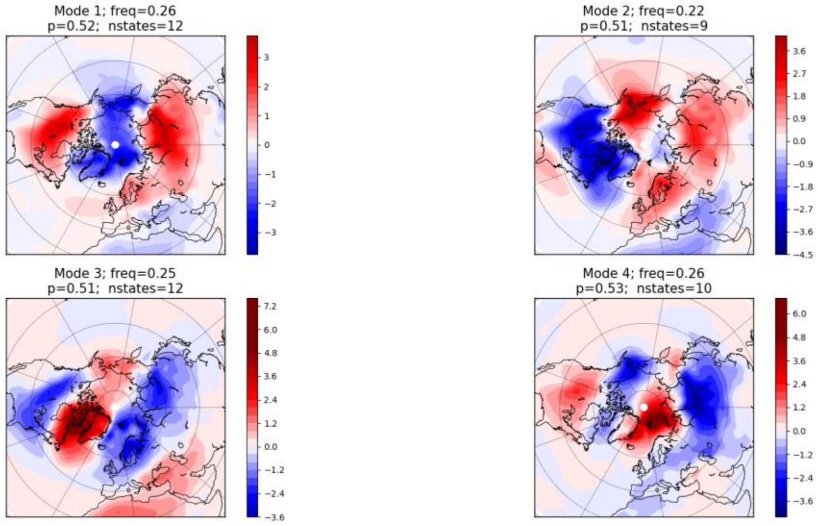
Probability of transitions to given regime from other regimes



HGT composites:



SAT composites:



Distance

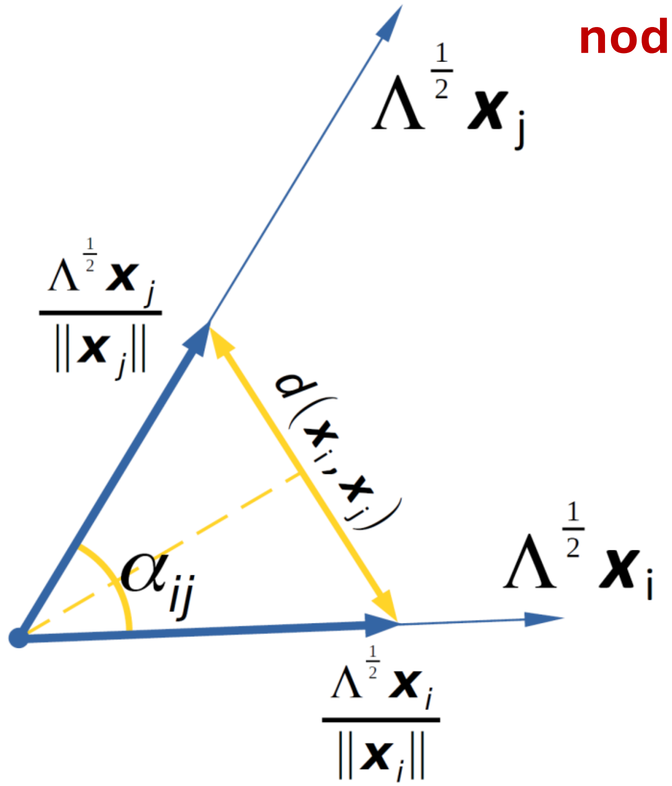
We use metric

$$d(\mathbf{x}_i, \mathbf{x}_j) = \left\| \frac{\mathbf{x}_i}{\|\mathbf{x}_i\|} - \frac{\mathbf{x}_j}{\|\mathbf{x}_j\|} \right\| = 2\left|\sin\frac{\alpha_{ij}}{2}\right|$$

$$\|\mathbf{x}\| = (\mathbf{x}^T \Lambda \mathbf{x})^{\frac{1}{2}}$$

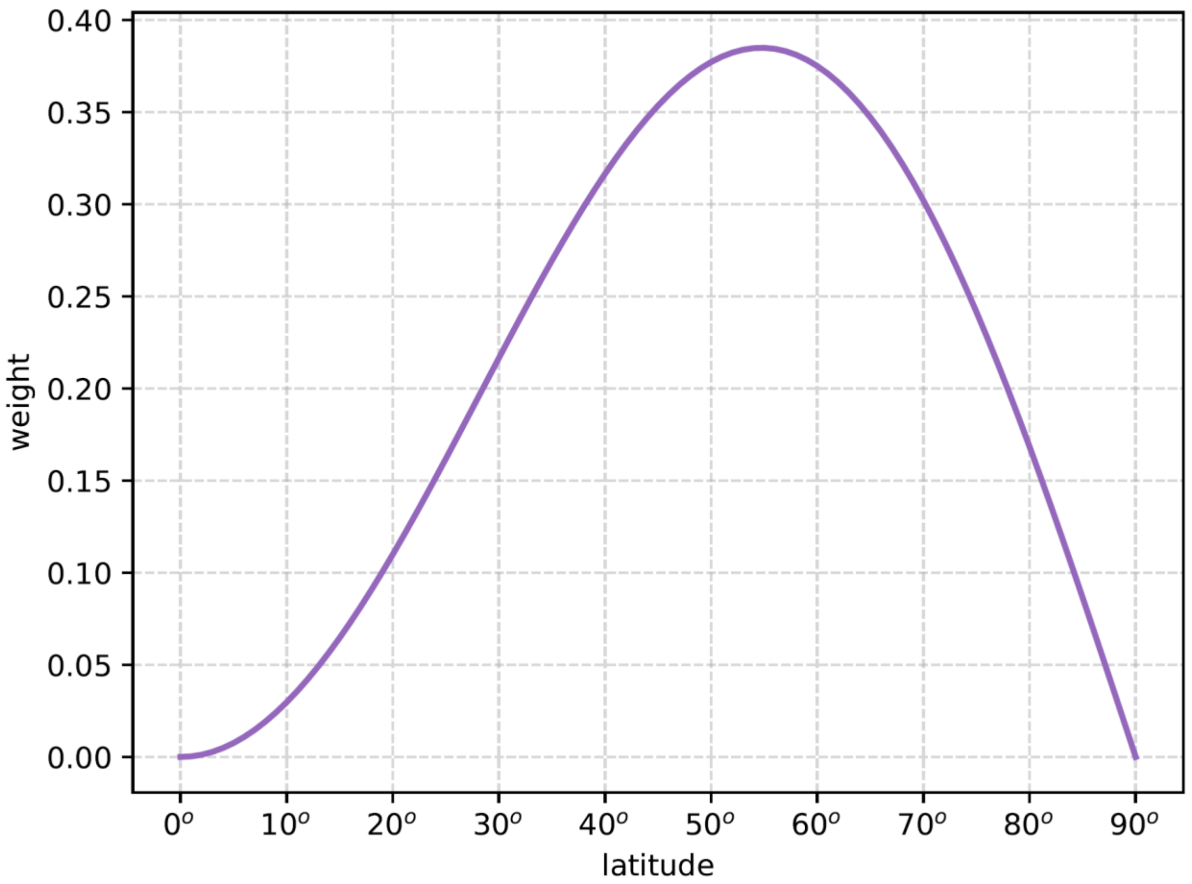
$$\boldsymbol{\Lambda} = \begin{pmatrix} \Lambda_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Lambda_{NN} \end{pmatrix}$$

- weights of grid nodes



$$\Lambda_{mm} = \cos\theta_m \sin^2\theta_m$$

- Latitudinal dependence



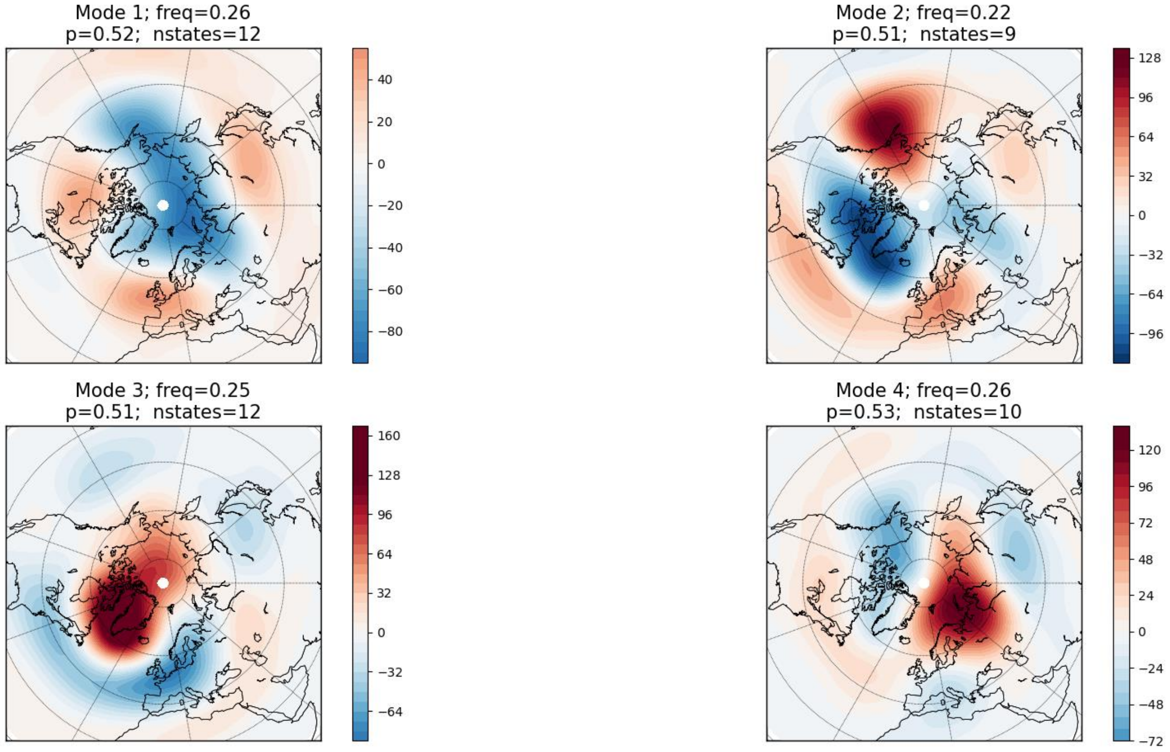
Hidden Markov model

Composite pattens

Reanalysis data: HGT-500, 1950-2023, winters **(NCEP/NCAR reanalysis)**

mean observed pattern given a community A_k:

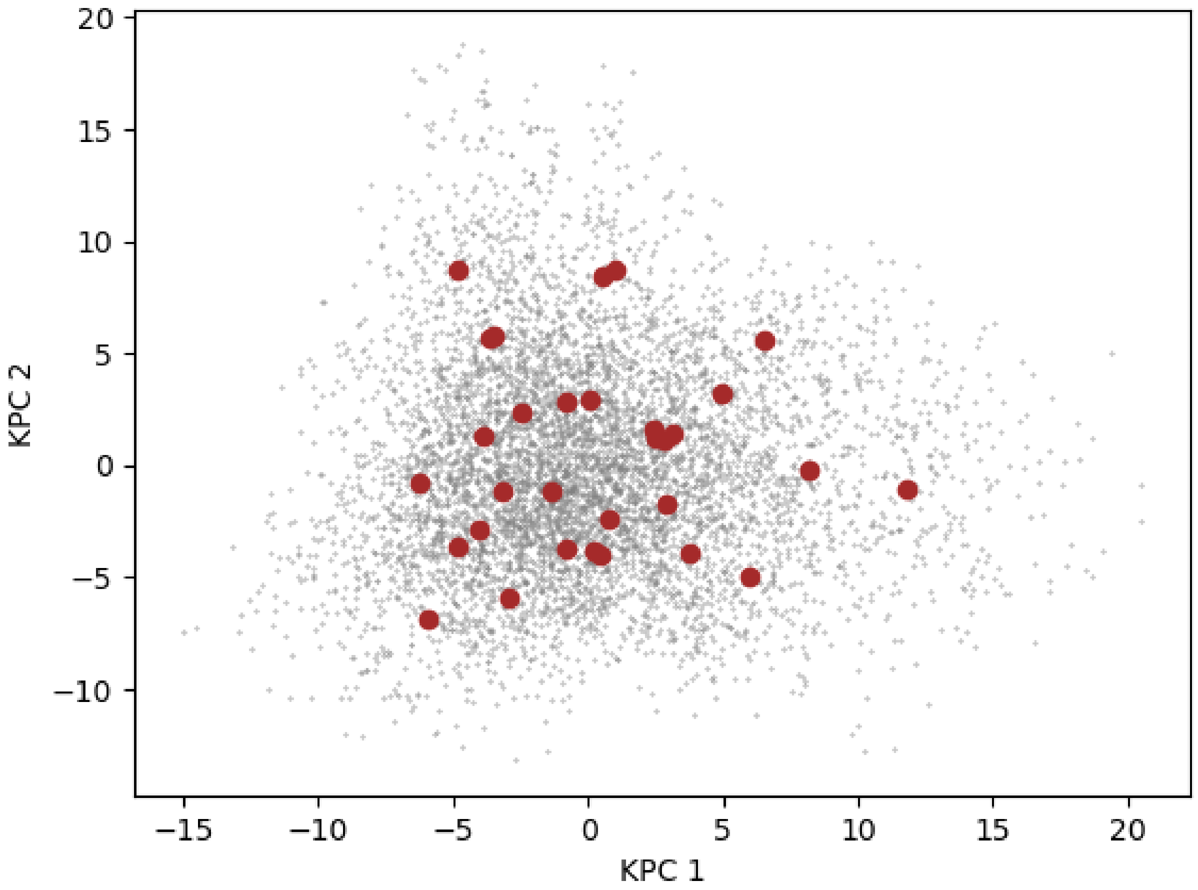
$$\langle Y_t | A_k \rangle = \frac{\sum_t Y_t \gamma_t(A_k)}{\sum_t \gamma_t(A_k)}$$



Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)

Two leading KPC projection

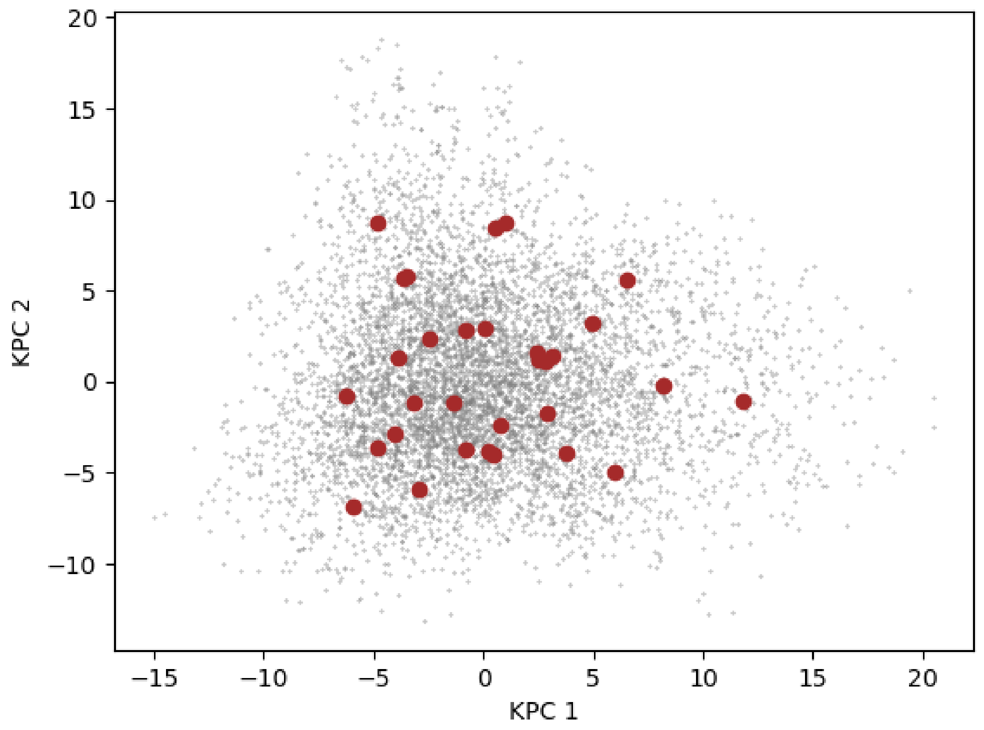
centers of hidden states are shown



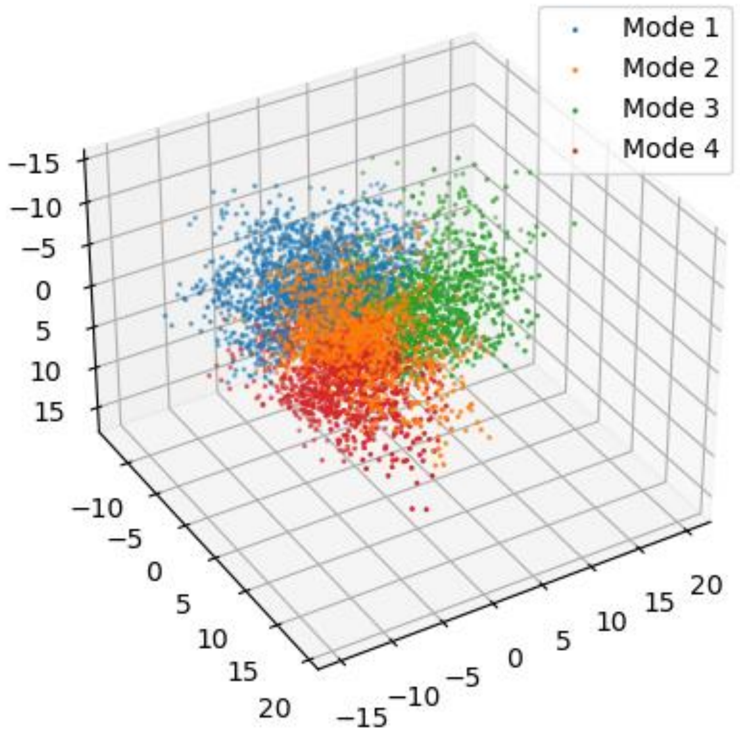
Hidden Markov model

Finding metastable states

Belonging of observed measurements to communities (in the space of 3 leading KPCs):

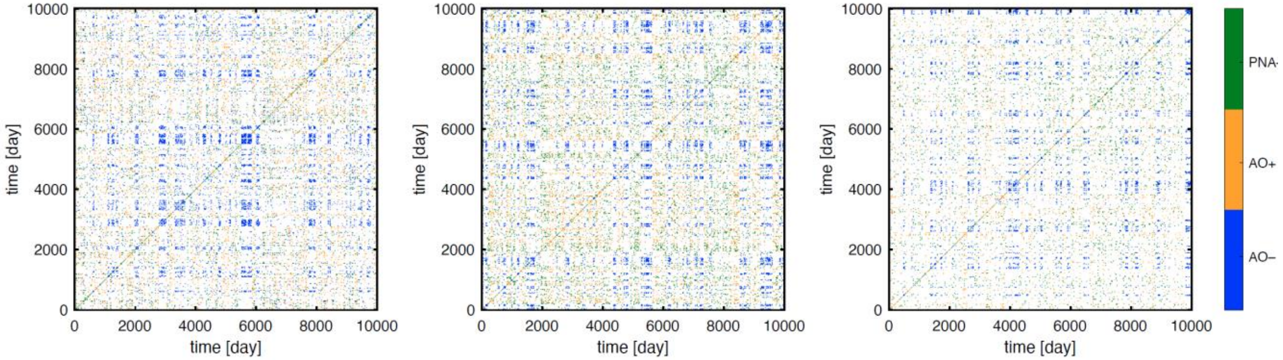


$$\gamma_t(A_k) = \sum_{i \in A_k} \gamma_t(X_i)$$

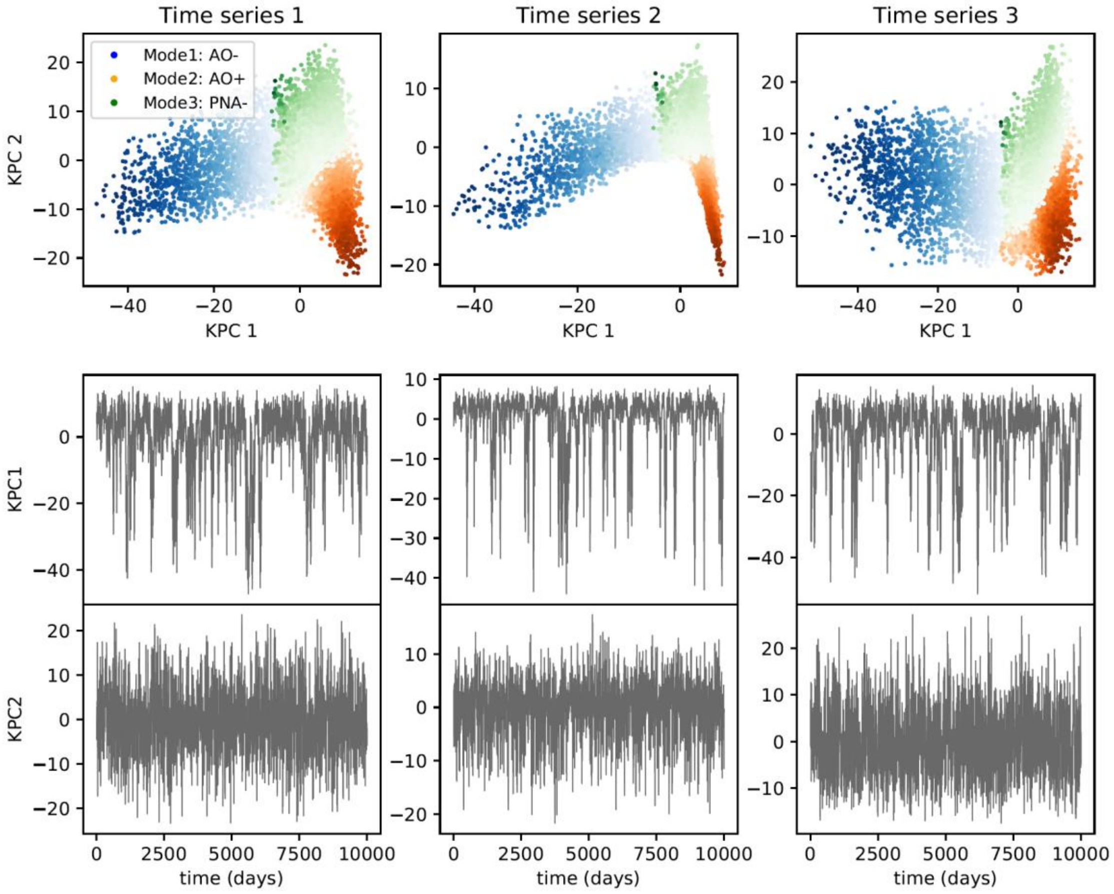


- **Quasigeostrophic 3-layer (QG3) model of the winter North Hemisphere atmosphere.**
- **Data is the stream function field anomalies from the middle layer (500 hPa).**
- **Here we show 3 time series of 10,000 days duration each.**

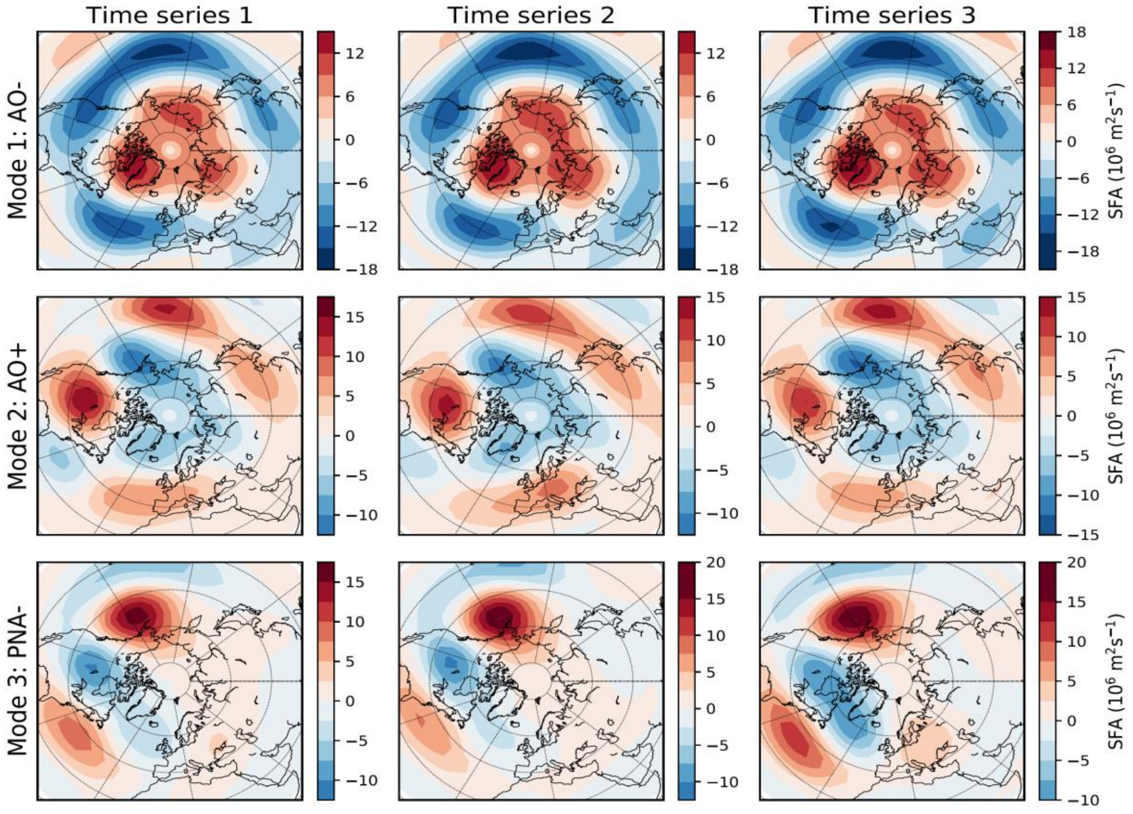
Recurrence plots



Modes in the kernel PC space



Composite patterns
(means over 20% most central points for each mode)



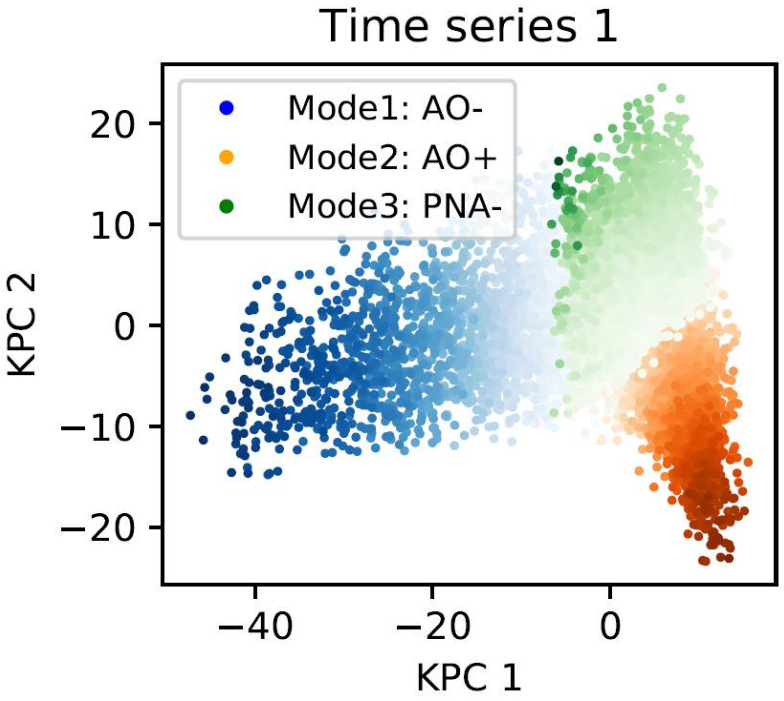
Kernel PCA

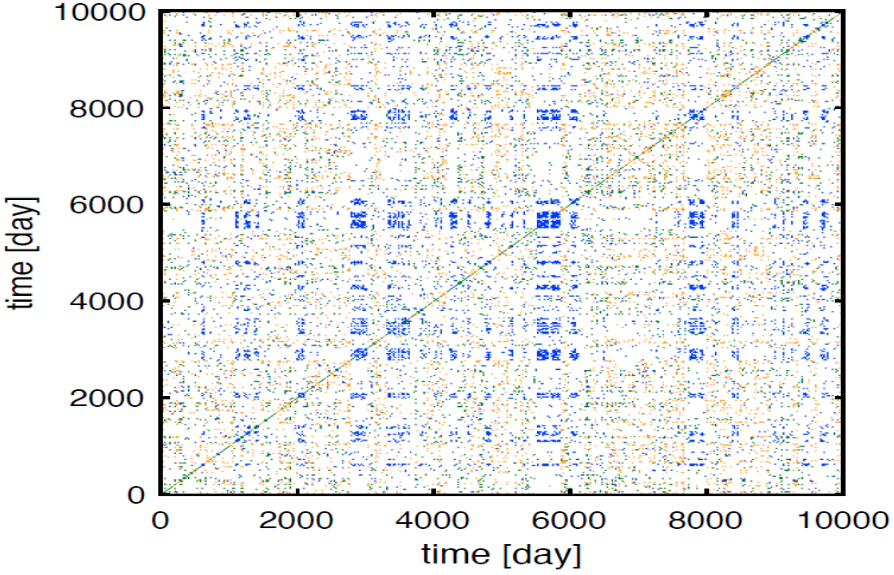
$$\mathbf{K}_{ij} = \exp\left\{-\frac{\boldsymbol{d}(\mathbf{x}_i,\ \mathbf{x}_j)^2}{2a^2}\right\}$$

Recurrence network

$$\mathbf{R}_{ij} = \begin{cases} 1, \mathbf{K}_{ij} > \varepsilon \\ 0, \text{otherwise} \end{cases}$$

Here we use metric $\boldsymbol{d}(\mathbf{x}_i,\ \mathbf{x}_j) = \left\| \mathbf{x}_i / \|\mathbf{x}_i\| - \mathbf{x}_j / \|\mathbf{x}_j\| \right\|$





Circulation of the mid-latitude atmosphere:

Weather regimes

Nonlinear theory: the regimes are associated with metastable states in the phase space

Example:

3-layer quasigeostrophic model

$$\frac{\partial q_1}{\partial t} = -J(\psi_1, q_1) + D_1(\psi_1, \psi_2) + S_1\,,$$

$$\frac{\partial q_2}{\partial t} = -J(\psi_2, q_2) + D_2(\psi_1, \psi_2, \psi_3) + S_2\,,$$

$$\frac{\partial q_3}{\partial t} = -J(\psi_3, q_3) + D_3(\psi_2, \psi_3) + S_3\,,$$

More than 100 positive Lyapunov exponents

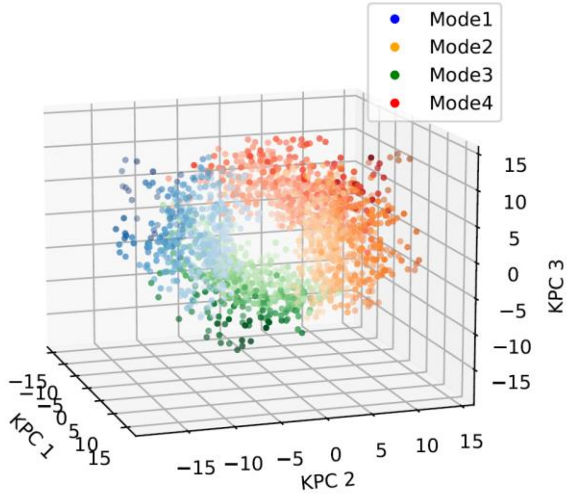
Vannitsem, S., & Nicolis, C. (1997). Lyapunov Vectors and Error Growth Patterns in a T21L3 Quasigeostrophic Model, *Journal of the Atmospheric Sciences*, *54*(2), 347-361

Unstable periodic orbits

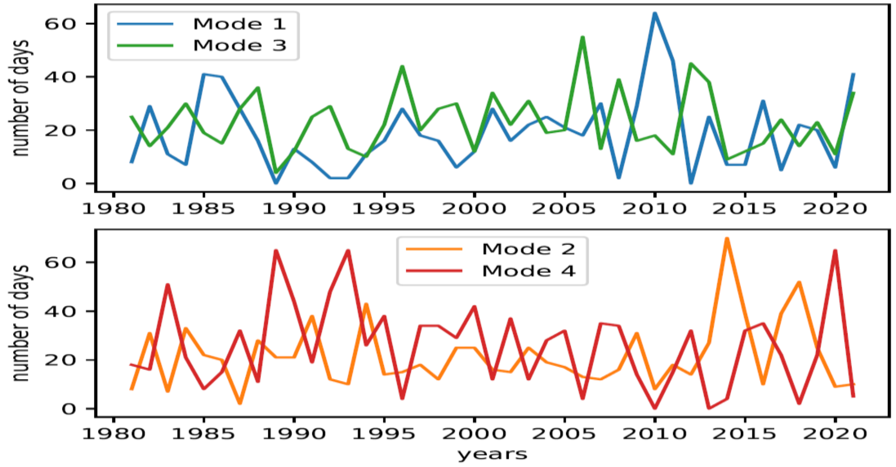
Andrey Gritsun, Valerio Lucarini,
Fluctuations, response, and resonances in a simple atmospheric model, Physica D: Nonlinear Phenomena, Volume 349, 2017

Reanalysis data: HGT-500, 1981-2019, winters (NCEP/NCAR reanalysis)

Modes in the kernel PC space

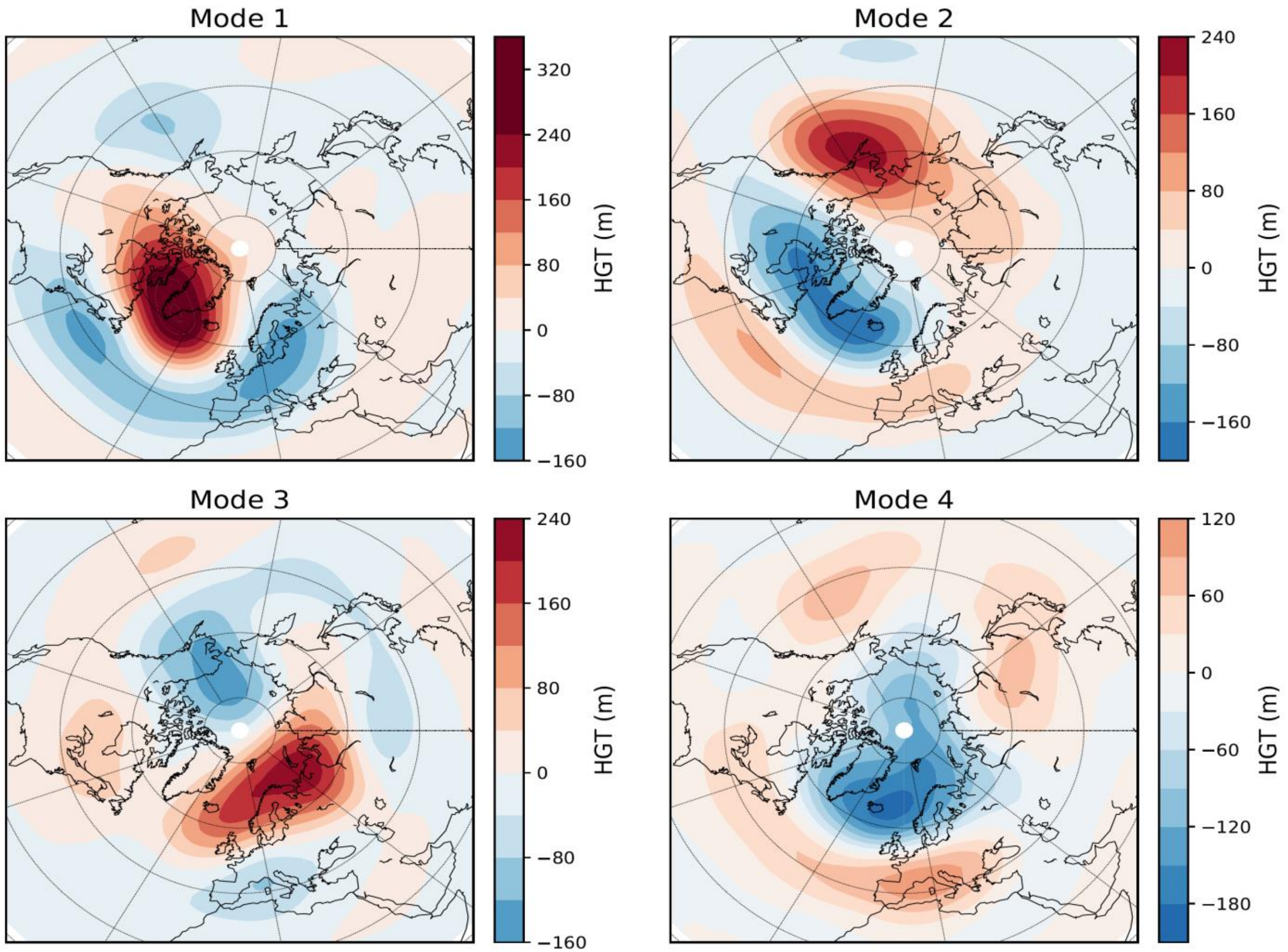


Days per winter



Composite patterns

(means over 20% most central points for each mode)



Problems:

- Detecting clusters in phase space is an incorrect problem
- Selecting a proper projection (subspace) of the phase space
- Both the dimension of a subspace and number of clusters are unknown
- The clusters may be non-Gaussian

Note on non-Markovian dynamics

Model with memory can be represented as Markov model in extended space

$$\mathbf{U}(t_{k+1}) = \mathbf{U}(t_k + \tau),$$

$$\mathbf{U}(t_{k+1} + \tau) = \mathbf{U}(t_k + 2\tau),$$

$$\vdots$$

$$\mathbf{U}[t_{k+1} + (m-1)\tau] = \mathbf{f}\{\mathbf{U}(t_k), \mathbf{U}(t_k + \tau), \ldots, \mathbf{U}[t_k + (m-1)\tau]\}$$

Reduced matrix :

$$\overline{Q}_{kl} = \frac{\sum\limits_{i \in A_k, j \in A_l} Q_{ij} \pi_j}{\sum\limits_{j \in A_l} \pi_j}$$

Expected lifetime of regime k:

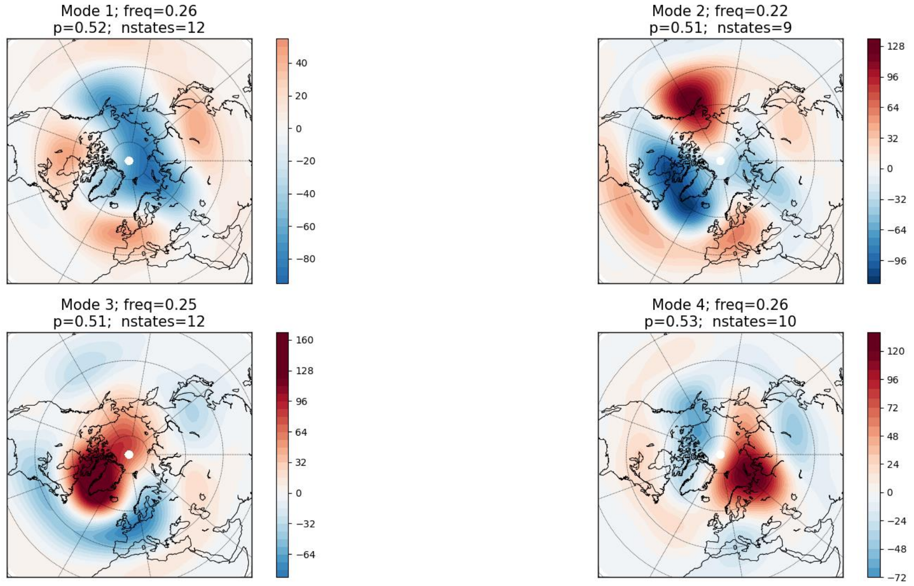
$$\overline{Q}_{kk} / (1 - \overline{Q}_{kk})$$

Hidden Markov model

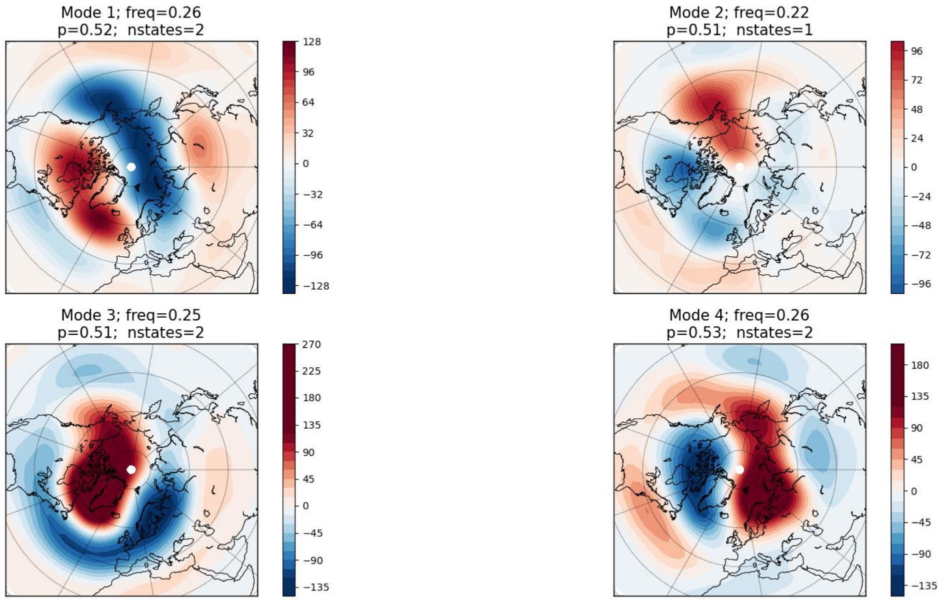
Composite pattens

Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)

Composite of all states associated with the regime:



The most stable hidden state of the regime:

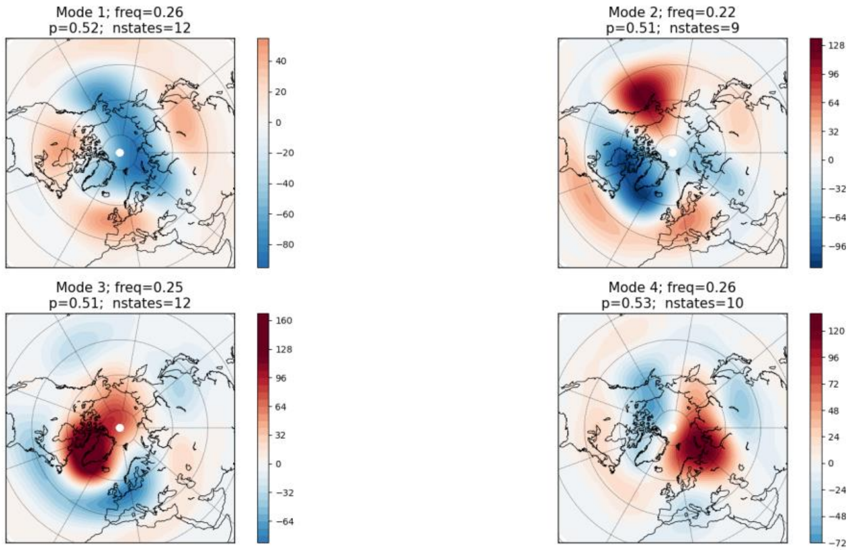


Hidden Markov model

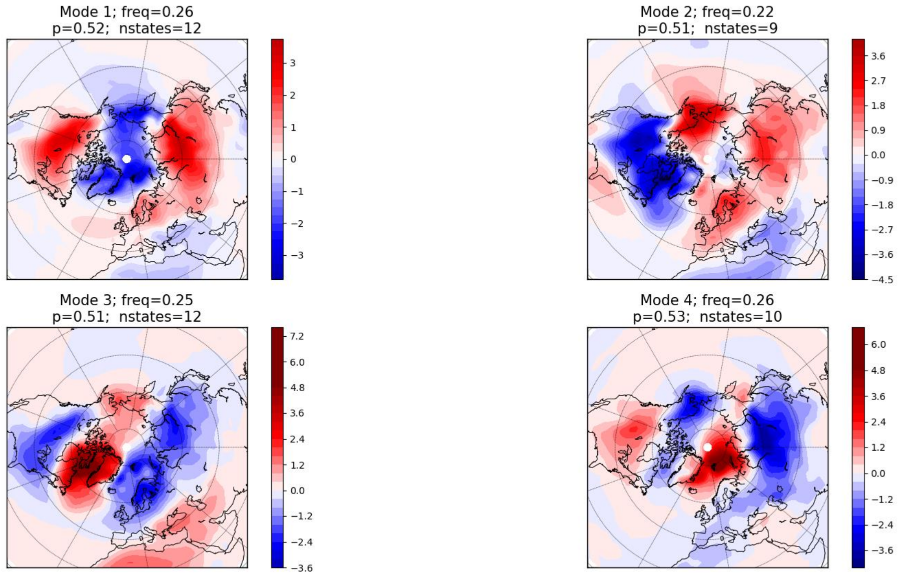
Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)

Impact to surface atmosphere temperatures (SAT)

HGT composites:



SAT composites:

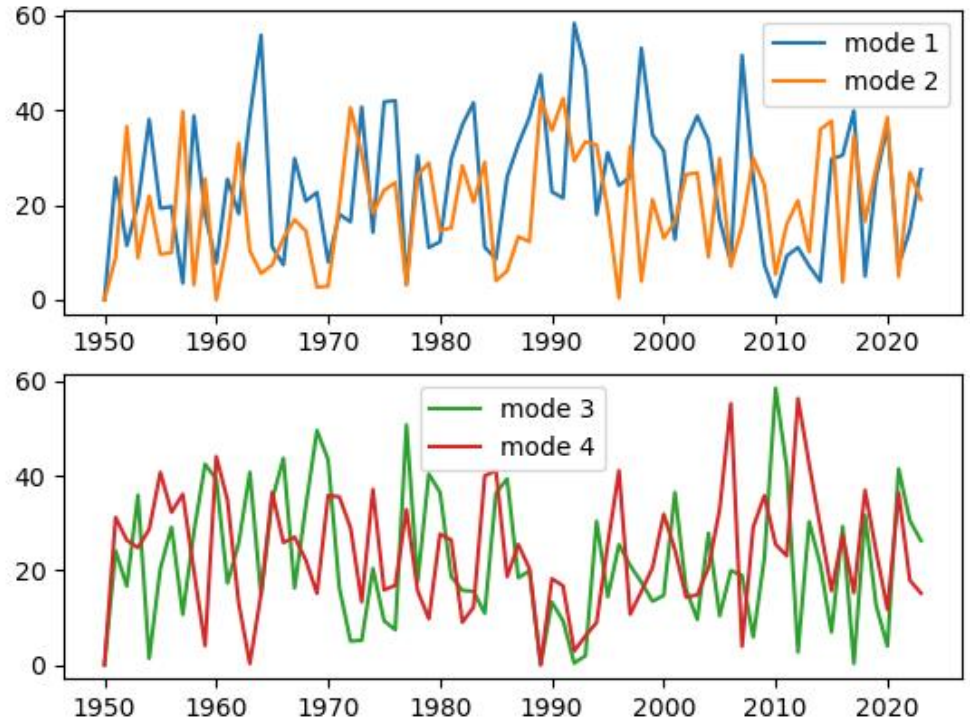


Hidden Markov model

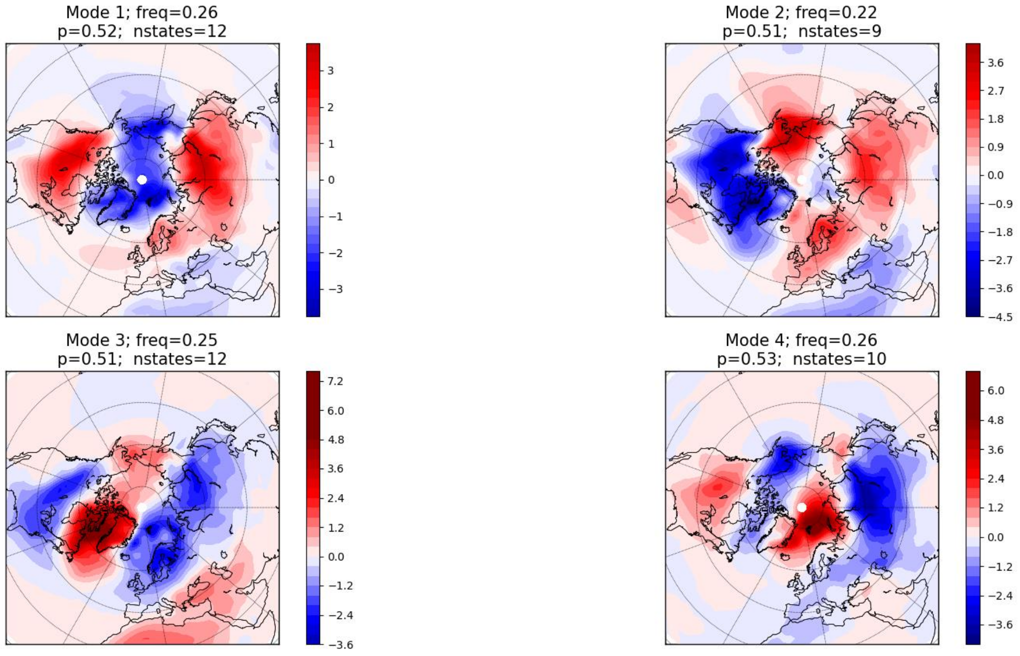
Reanalysis data: HGT-500, 1950-2023, winters (NCEP/NCAR reanalysis)

Impact to surface atmosphere temperatures (SAT)

Regime frequencies (number of days per winter:



SAT composites:



Hidden Markov model