

Вихревые модели плоских турбулентных течений, ограниченных стенками

В.Л. Миронов, С.В. Миронов

**Институт физики микроструктур РАН
Нижний Новгород**

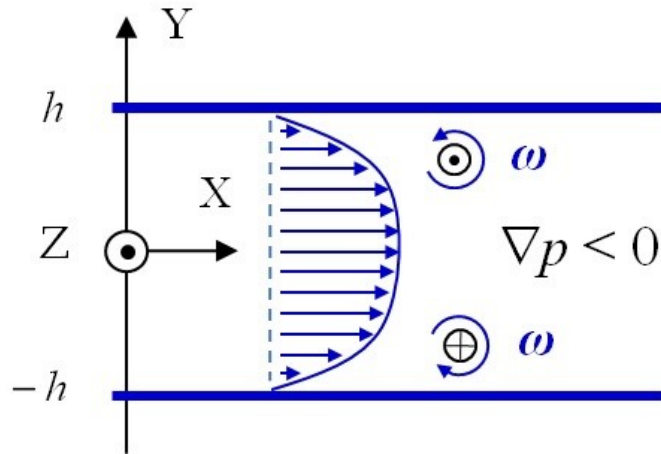
Турбулентность, динамика атмосферы и климата, 2024

**Моделирование распределения
скорости плоского полностью
развитого стационарного
турбулентного потока
в прямоугольном канале**

План

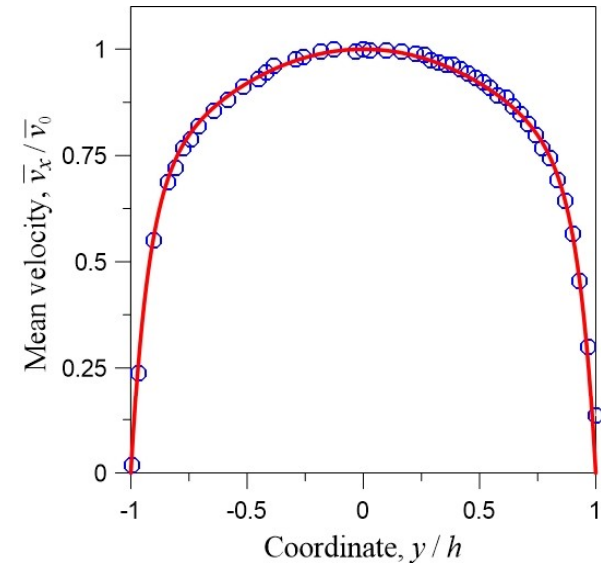
- **Модель вихревого потока**
- **Уравнения плоского турбулентного потока**
- **Сравнение с экспериментальными данными**
- **Заключение**

АНОНС



$$\bar{v}_x = \sigma \frac{\text{ch}(h/\lambda) - \text{ch}(y/\lambda)}{\text{sh}(h/\lambda)} + \frac{(1-\sigma)h}{2\lambda} \left(1 - \frac{y^2}{h^2} \right),$$

λ, σ

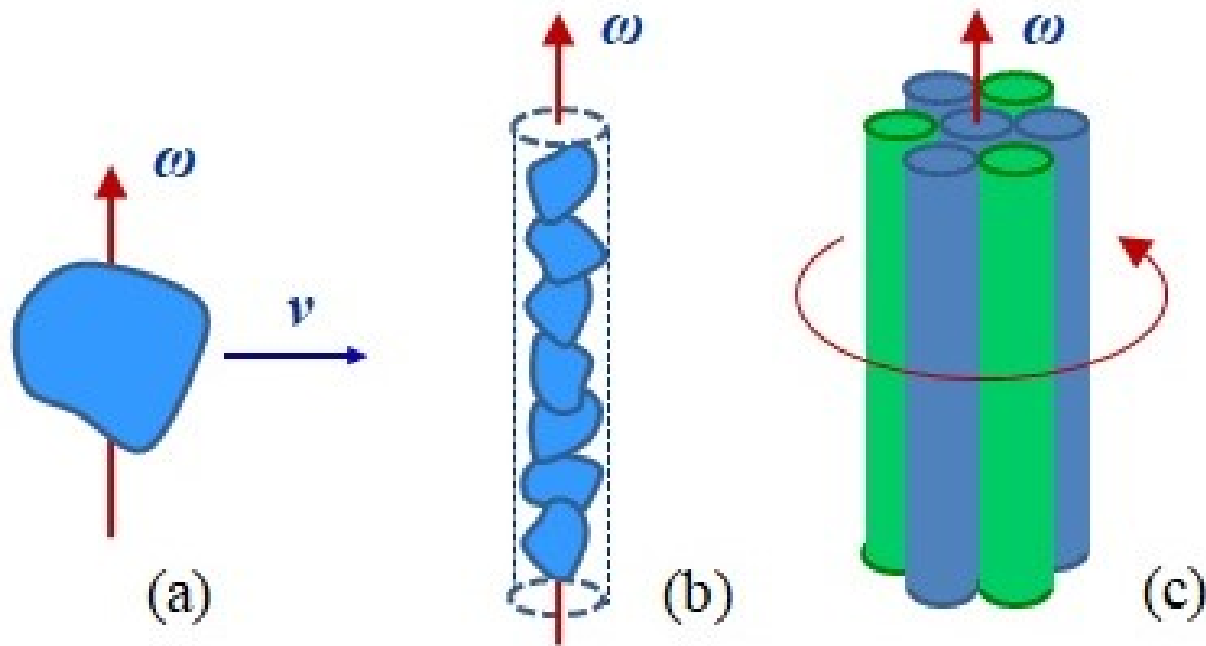


E.M. Thurlow, et al. (2000)

V.L. Mironov, S.V. Mironov, Vortex model of plane turbulent air flows in channels, *Advances in Aerodynamics*, **6**(8) 1-17 (**2024**).

Модель вихревого потока

Вихревые трубки



H. Helmholtz, "Über Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen", *Journal für die Reine und Angewandte Mathematik*, **55**, 25—55 (**1858**).

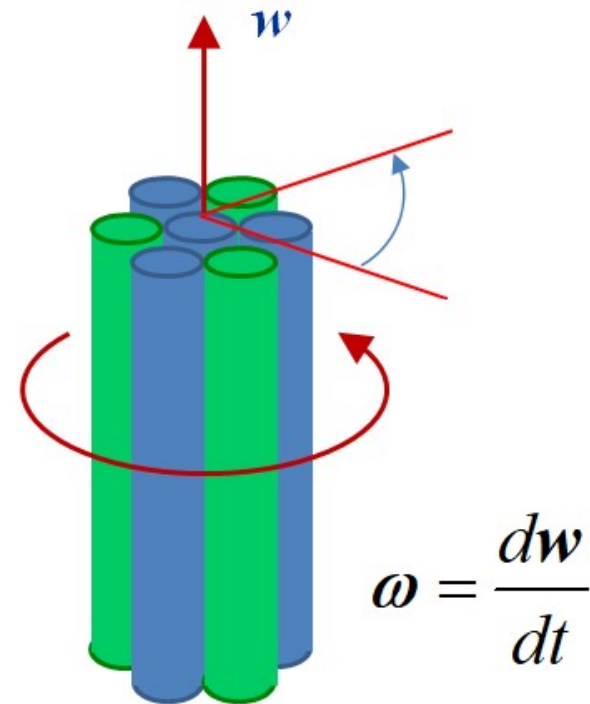
Уравнения вихревого потока

$$\frac{1}{c} \left(\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) - \mu \Delta \right) \mathbf{v} + \nabla \times \mathbf{w} + \nabla u = 0,$$

$$\frac{1}{c} \left(\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) - \mu \Delta \right) u + \nabla \cdot \mathbf{v} = 0,$$

$$\frac{1}{c} \left(\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) - \mu \Delta \right) \mathbf{w} - \nabla \times \mathbf{v} + \nabla \xi = 0,$$

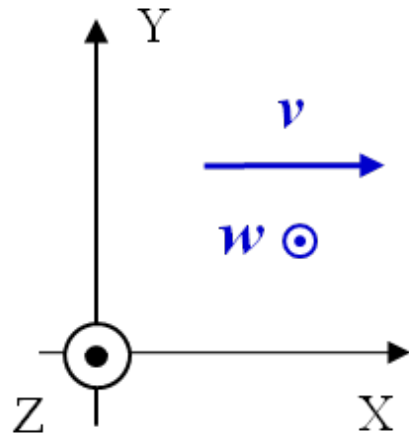
$$\frac{1}{c} \left(\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) - \mu \Delta \right) \xi + \nabla \cdot \mathbf{w} = 0.$$



V.L. Mironov, S.V. Mironov, Generalized sedeonic equations of hydrodynamics, European Physical Journal Plus, **135**(9), 708 (**2020**).

Плоское вихревое течение

Плоское течение



$$v_x = v_x(y, t)$$

$$w_z = w_z(y, t)$$

$$\frac{1}{c} \frac{\partial v_x}{\partial t} - \frac{\mu}{c} \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial w_z}{\partial y} - g = 0,$$

$$\frac{1}{c} \frac{\partial w_z}{\partial t} - \frac{\mu}{c} \frac{\partial^2 w_z}{\partial y^2} + \frac{\partial v_x}{\partial y} = 0.$$

$$\frac{1}{c\rho} \frac{\partial p}{\partial x} = -g$$

Стационарный турбулентный ПОТОК

$$\mathbf{v}(\mathbf{r}, t) = \bar{\mathbf{v}}(\mathbf{r}) + \mathbf{v}'(\mathbf{r}, t) \quad \mathbf{w}(\mathbf{r}, t) = \bar{\mathbf{w}}(\mathbf{r}) + \mathbf{w}'(\mathbf{r}, t)$$

$$\bar{a}(y) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T a(y, t) dt,$$

$$w_z(y, t) = 2c\omega_z(y)t + \bar{\varphi}_z(y) + \varphi'_z(y, t).$$

$$\begin{aligned} -\frac{\mu}{c} \frac{\partial^2 \bar{v}_x}{\partial y^2} + \frac{1}{c} \frac{\partial}{\partial y} \overline{v'_x v'_y} + \frac{\partial \bar{\varphi}_z}{\partial y} - g &= 0, \\ -\frac{\mu}{c} \frac{\partial^2 \bar{\varphi}_z}{\partial y^2} + \frac{1}{c} \frac{\partial}{\partial y} \overline{\varphi'_z v'_y} + \frac{\partial \bar{v}_x}{\partial y} - 2\omega_z(y) &= 0. \end{aligned}$$

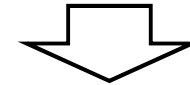
Гипотеза Буссинеска

$$-\overline{v'_x v'_y} = \mu_T \frac{\partial \bar{v}_x}{\partial y}, \quad -\frac{\mu}{c} \frac{\partial^2 \bar{v}_x}{\partial y^2} + \frac{1}{c} \frac{\partial}{\partial y} \overline{v'_x v'_y} + \frac{\partial \bar{\varphi}_z}{\partial y} - g = 0,$$

$$-\overline{\varphi'_z v'_y} = \mu_T \frac{\partial \bar{\varphi}_z}{\partial y}, \quad -\frac{\mu}{c} \frac{\partial^2 \bar{\varphi}_z}{\partial y^2} + \frac{1}{c} \frac{\partial}{\partial y} \overline{\varphi'_z v'_y} + \frac{\partial \bar{v}_x}{\partial y} - 2\omega_z(y) = 0.$$

$$\mu_T = \text{const}$$

$$\lambda = \frac{\mu + \mu_T}{c} = \text{const}$$



$$-\lambda \frac{\partial^2 \bar{v}_x}{\partial y^2} + \frac{\partial \bar{\varphi}_z}{\partial y} - g = 0,$$

$$-\lambda \frac{\partial^2 \bar{\varphi}_z}{\partial y^2} + \frac{\partial \bar{v}_x}{\partial y} - 2\omega_z(y) = 0.$$

Стационарный плоский турбулентный поток

$$-\lambda \frac{\partial^2 \bar{v}_x}{\partial y^2} + \frac{\partial \bar{\varphi}_z}{\partial y} - g = 0,$$

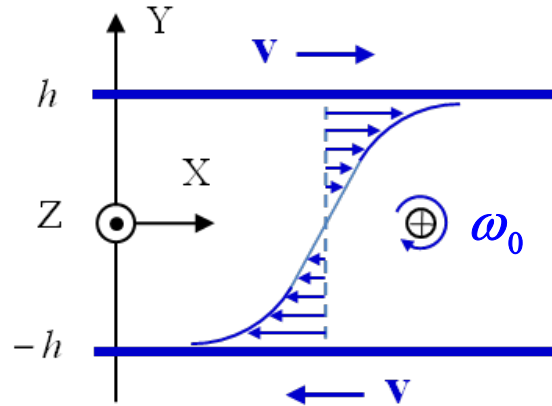
$$-\lambda \frac{\partial^2 \bar{\varphi}_z}{\partial y^2} + \frac{\partial \bar{v}_x}{\partial y} - 2\omega_z(y) = 0.$$

$$\lambda = \frac{\mu + \mu_T}{c}$$

V.L. Mironov, S.V. Mironov, Vortex model of plane turbulent air flows in channels, *Advances in Aerodynamics*, 6(8) 1-17 (2024).

Сравнение с экспериментальными данными и моделированием

Турбулентное течение Куэтта



$$\omega_z(y) = \omega_0$$

$$-\lambda \frac{\partial^2 \bar{v}_x}{\partial y^2} + \frac{\partial \bar{\varphi}_z}{\partial y} = 0,$$

$$-\lambda \frac{\partial^2 \bar{\varphi}_z}{\partial y^2} + \frac{\partial \bar{v}_x}{\partial y} - 2\omega_0 = 0.$$

$$\bar{v}_x(h) = v,$$

$$\bar{v}_x(-h) = -v,$$

$$\bar{\varphi}_z(h) = 0,$$

$$\bar{\varphi}_z(-h) = 0.$$

$$\bar{v}_x = \alpha v \frac{y}{h} + (1 - \alpha) v \frac{\text{sh}(y/\lambda)}{\text{sh}(h/\lambda)},$$

$$\alpha = 2\omega_0 h / v.$$

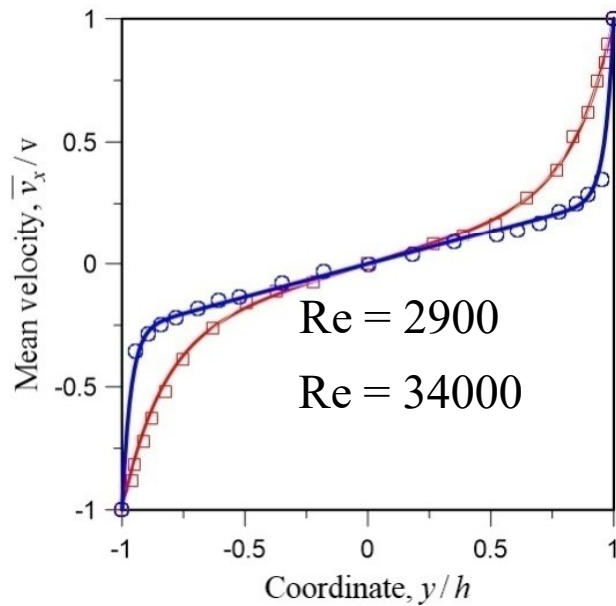
$$\bar{\varphi}_z = (1 - \alpha) v \frac{\text{ch}(y/\lambda) - \text{ch}(h/\lambda)}{\text{sh}(h/\lambda)}.$$

$$\lambda = \frac{\mu + \mu_T}{c}$$

V.L. Mironov, S.V. Mironov, Vortex model of plane turbulent air flows in channels, *Advances in Aerodynamics*, 6(8) 1-17 (2024).

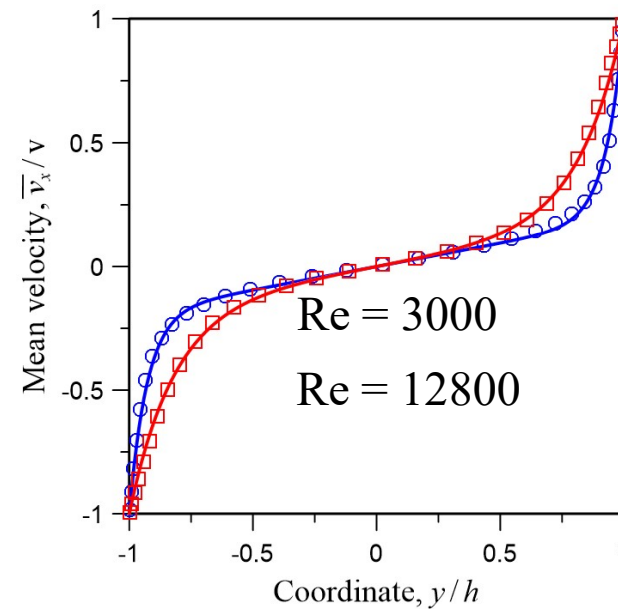
Турбулентное течение Куэтта

Experiment



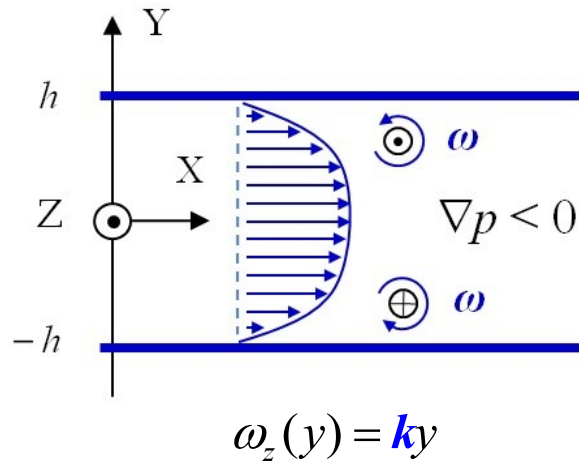
H. Reichardt (1956).

DNS results



T. Tsukahara, et al. (2006)

Турбулентное течение Пуазейля



$$-\lambda \frac{\partial^2 \bar{v}_x}{\partial y^2} + \frac{\partial \bar{\varphi}_z}{\partial y} - g = 0,$$

$$\bar{v}_x(h) = 0,$$

$$\bar{v}_x(-h) = 0,$$

$$-\lambda \frac{\partial^2 \bar{\varphi}_z}{\partial y^2} + \frac{\partial \bar{v}_x}{\partial y} + 2ky = 0.$$

$$\bar{\varphi}_z(h) = 0,$$

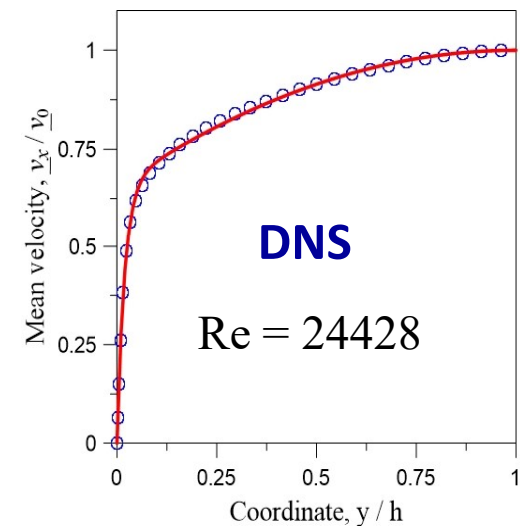
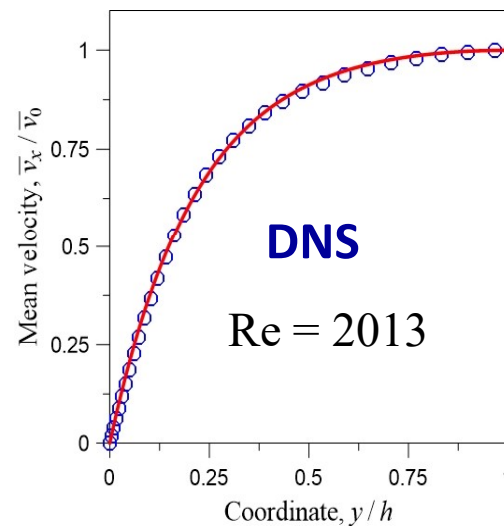
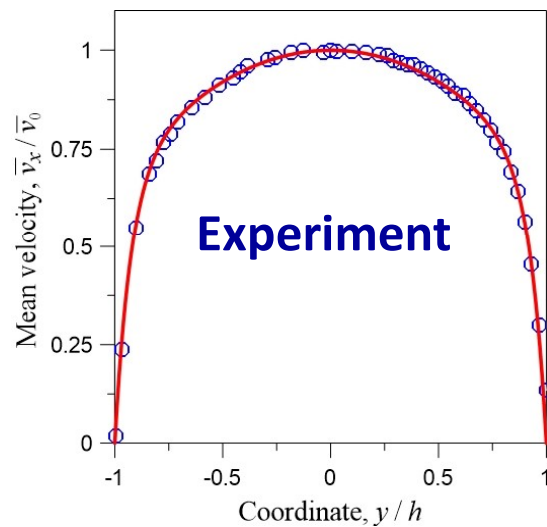
$$\bar{\varphi}_z(-h) = 0.$$

$$\bar{v}_x = \sigma gh \frac{\text{ch}(h/\lambda) - \text{ch}(y/\lambda)}{\text{sh}(h/\lambda)} + gh^2 \frac{(1-\sigma)}{2\lambda} \left(1 - \frac{y^2}{h^2} \right), \quad \lambda = \frac{\mu + \mu_T}{c}$$

$$\bar{\varphi}_z = -\sigma gh \left(\frac{\text{Sh}(y/\lambda)}{\text{Sh}(h/\lambda)} - \frac{y}{h} \right) \quad \sigma = 1 - \frac{2\lambda k}{g}$$

V.L. Mironov, S.V. Mironov, Vortex model of plane turbulent air flows in channels, *Advances in Aerodynamics*, **6**(8) 1-17 (2024).

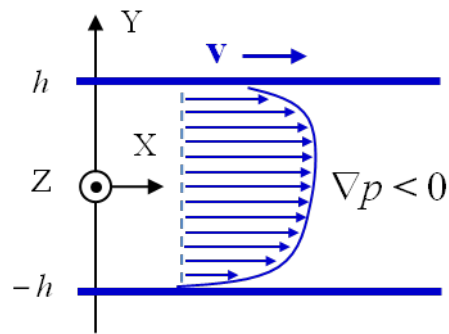
Турбулентное течение Пуазейля



E.M. Thurlow, et al. (2000)

T. Tsukahara, et al. (2006)

Течение Пуазейля - Куэтта



$$\omega_z(y) = -\omega_0 + ky$$

$$-\lambda \frac{\partial^2 \bar{v}_x}{\partial y^2} + \frac{\partial \bar{\varphi}_z}{\partial y} - g = 0,$$

$$-\lambda \frac{\partial^2 \bar{\varphi}_z}{\partial y^2} + \frac{\partial \bar{v}_x}{\partial y} - 2\omega_0 + 2ky = 0.$$

$$\bar{v}_x(h) = v,$$

$$\bar{v}_x(-h) = 0,$$

$$\bar{\varphi}_z(h) = 0,$$

$$\bar{\varphi}_z(-h) = 0.$$

$$v_x = \sigma gh \frac{\text{ch}(h/\lambda) - \text{ch}(y/\lambda)}{\text{sh}(h/\lambda)} + \frac{gh^2(1-\sigma)}{2\lambda} \left(1 - \frac{y^2}{h^2}\right) + v \frac{(1-\varepsilon)}{2} \frac{\text{sh}(y/\lambda)}{\text{sh}(h/\lambda)} + \frac{v}{2} \left(1 + \varepsilon \frac{y}{h}\right)$$

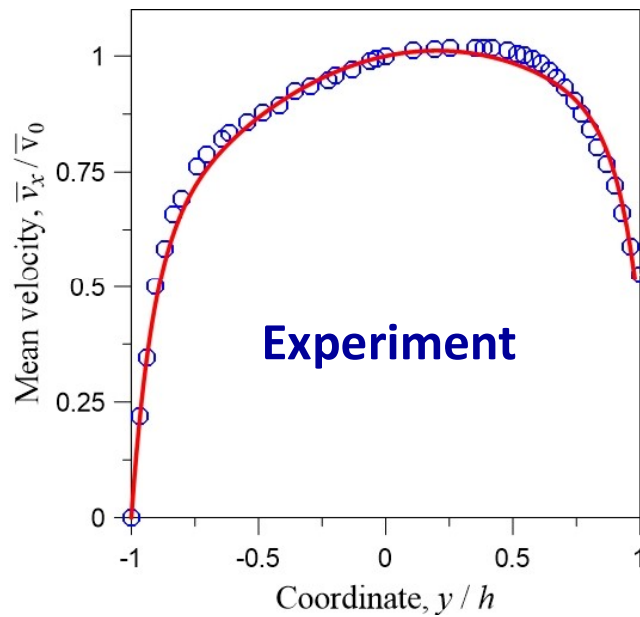
$$\lambda = \frac{\mu + \mu_T}{c}$$

$$\sigma = 1 - \frac{2\lambda k}{g}$$

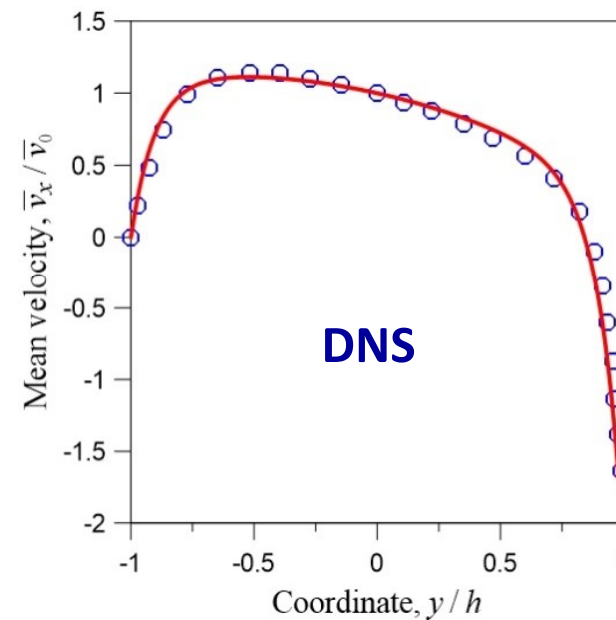
$$\varepsilon = \frac{2\omega_0 h}{v}$$

V.L. Mironov, S.V. Mironov, Vortex model of plane turbulent air flows in channels, *Advances in Aerodynamics*, **6**(8) 1-17 (**2024**).

Течение Пуазейля-Куэтта



E.M. Thurlow, et al. (2000)



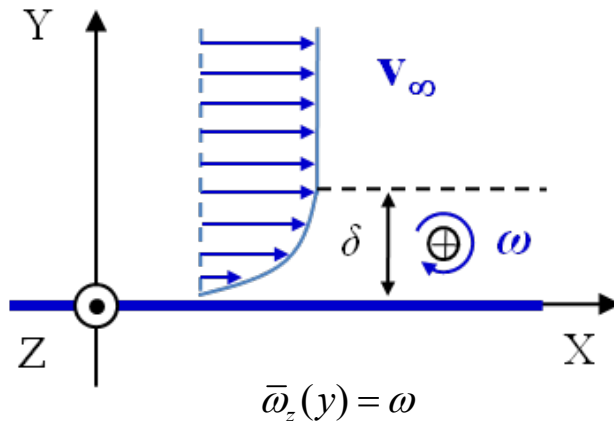
Q.T. Nguyen, et al. (2013)

Заключение

- Предложена простая модель полностью установившегося плоского турбулентного потока на основе уравнений для вихревой жидкости.
- Расчеты показывают хорошее согласие модельных профилей средней скорости в каналах с экспериментальными данными и результатами прямого численного моделирования.

**Спасибо
за внимание!**

Пристеночное течение



$$-\lambda \frac{\partial^2 \bar{v}_x}{\partial y^2} + \frac{\partial \bar{\varphi}_z}{\partial y} = 0,$$

$$-\lambda \frac{\partial^2 \bar{\varphi}_z}{\partial y^2} + \frac{\partial \bar{v}_x}{\partial y} - 2\omega = 0.$$

$$\bar{v}_x(0) = 0$$

$$\bar{v}_x(\delta) = v_\infty,$$

$$\bar{\varphi}_z(0) = 0,$$

$$\bar{\varphi}_z(\delta) = \varphi_\delta.$$

$$\bar{v}_x = v_\infty (1 - \beta) \frac{1 - \exp(-y/\lambda)}{1 - \exp(-\delta/\lambda)} + \beta y / \delta,$$

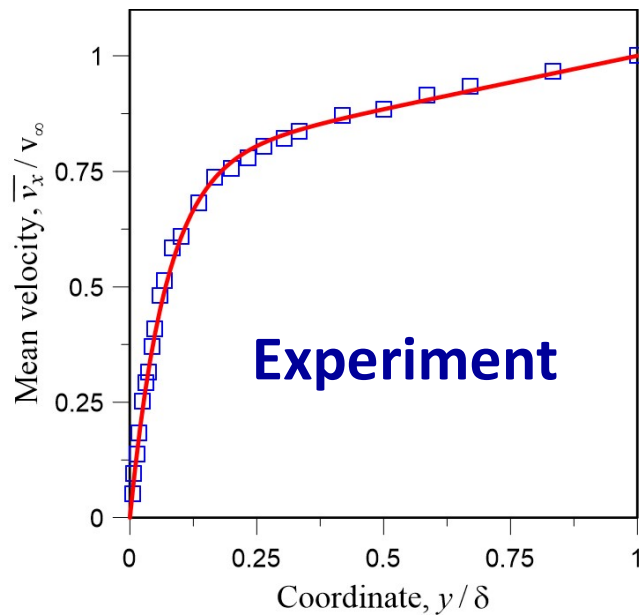
$$\beta = 2\omega \delta / v_\infty$$

$$\bar{\varphi}_z = \varphi_\delta \left(1 - \frac{1 - \exp(-y/\lambda)}{1 - \exp(-\delta/\lambda)} \right).$$

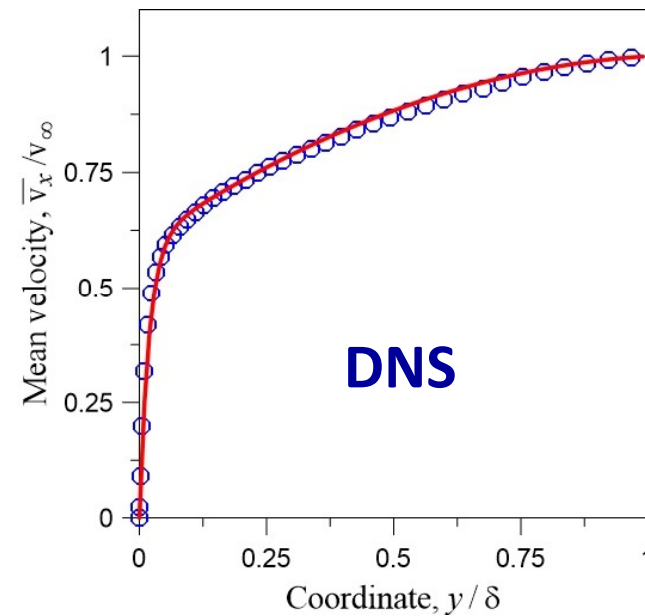
$$\lambda = \frac{\mu + \mu_T}{c}$$

V.L. Mironov, S.V. Mironov, Vortex model of plane turbulent air flows in channels, *Advances in Aerodynamics*, **6**(8) 1-17 (**2024**).

Пристеночное течение



Z. Gete, R.L. Evans (2003).



P. Schlatter, R. Orlu (2010)